Relativistic GNSS

– PECS project –

Univerza *v Ljubljani*
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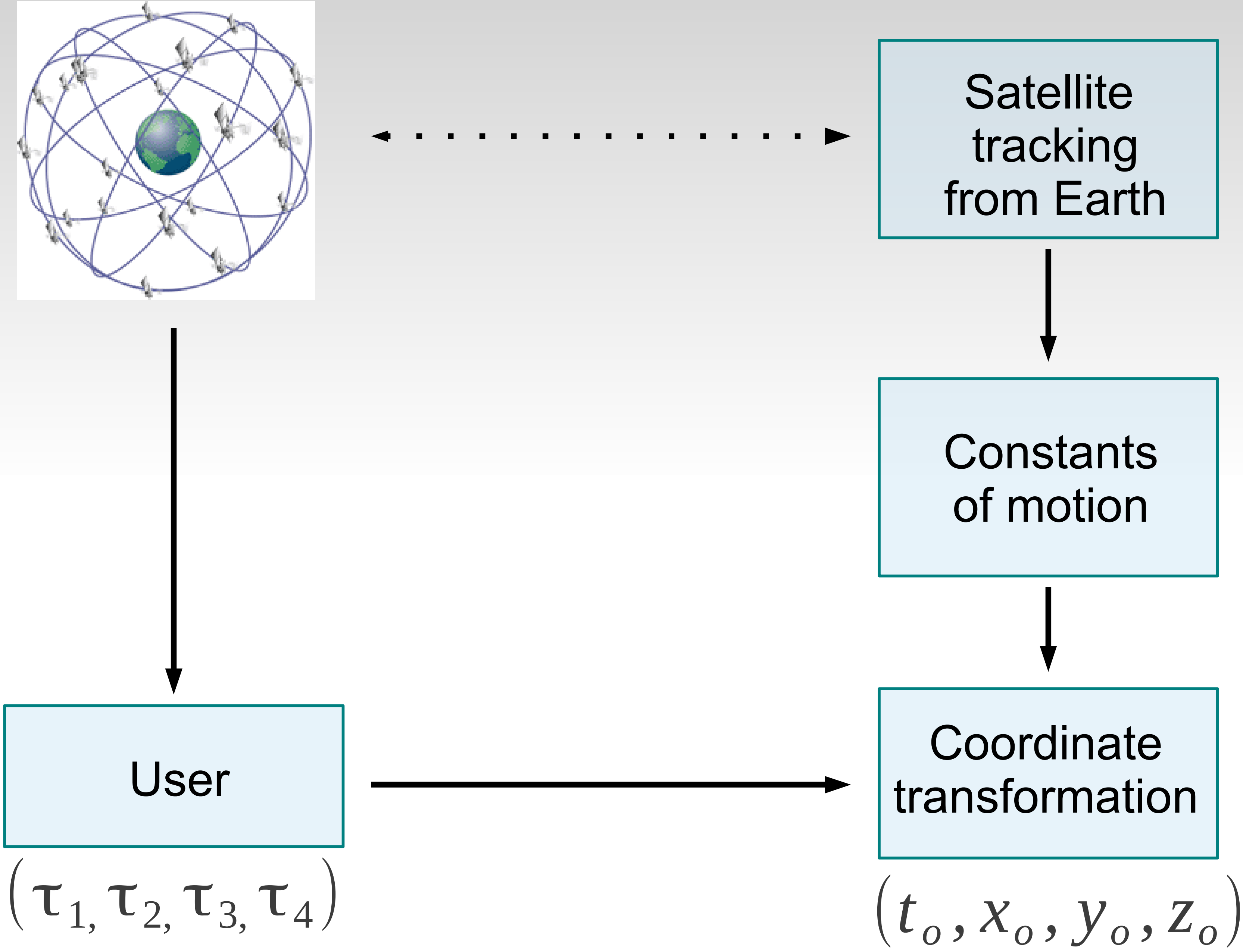


Satellite tracking from Earth

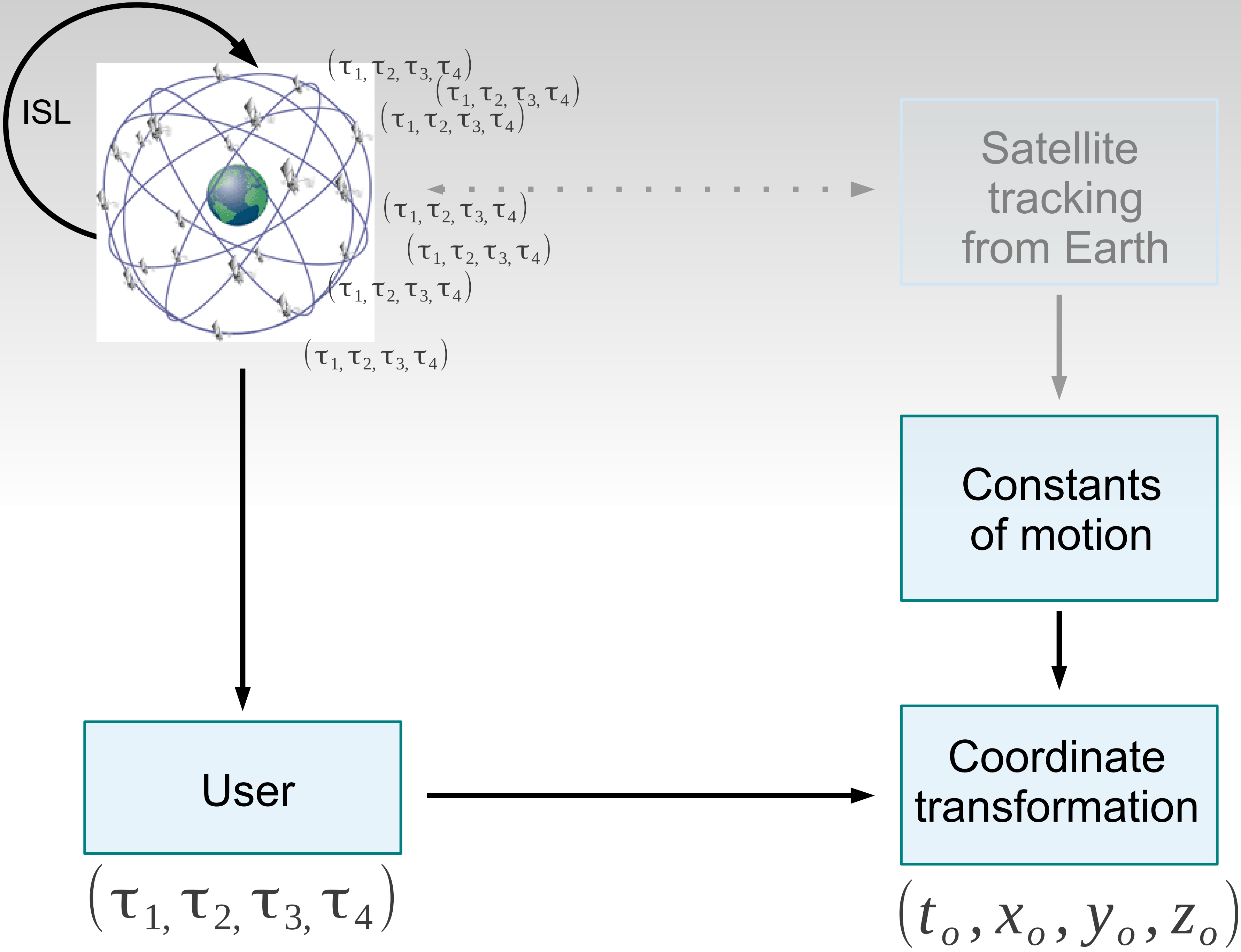
Constants of motion

Coordinate transformation

User

$(\tau_1, \tau_2, \tau_3, \tau_4)$

(t_o, x_o, y_o, z_o)



ISL

$(\tau_1, \tau_2, \tau_3, \tau_4)$

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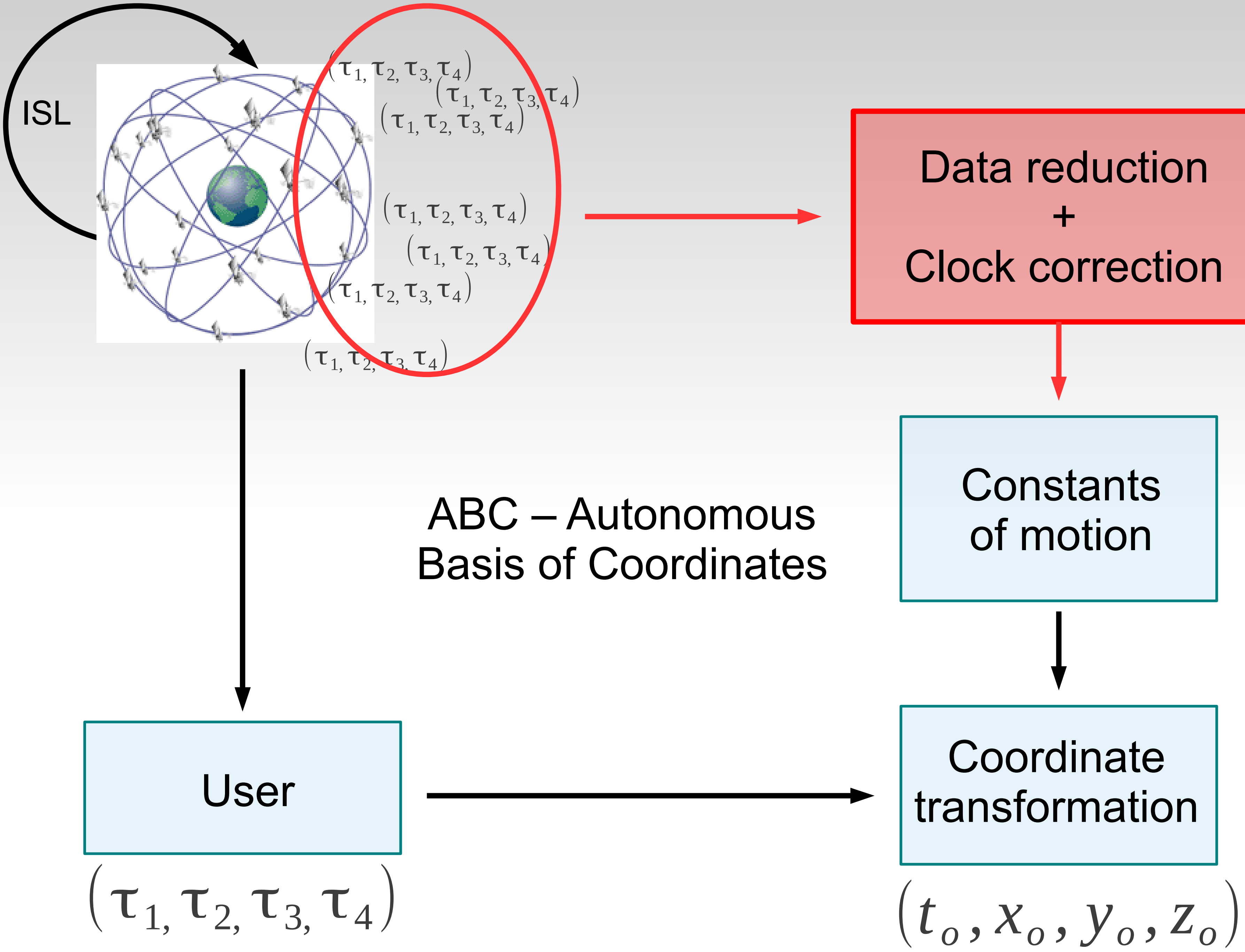

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Data reduction
+
Clock correction

Constants of motion

ABC – Autonomous Basis of Coordinates

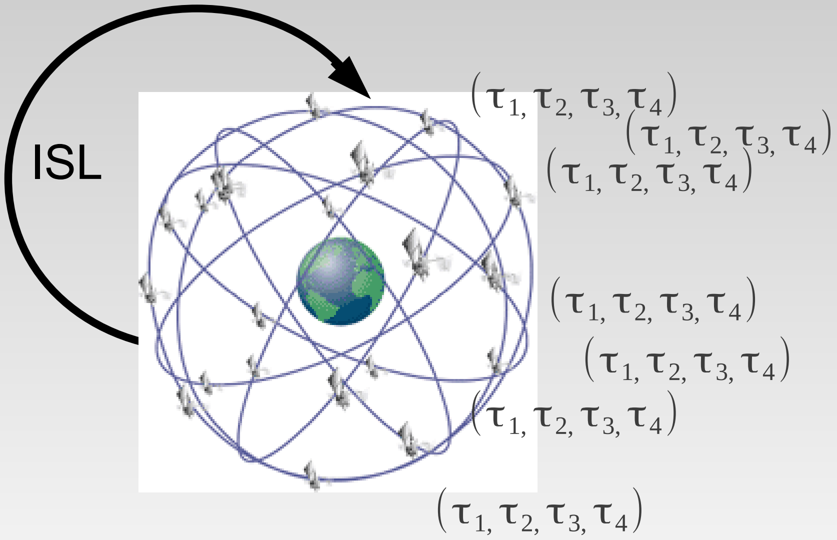
User

$(\tau_1, \tau_2, \tau_3, \tau_4)$

Coordinate transformation

(t_o, x_o, y_o, z_o)

University of Ljubljana, ESA Advanced Concepts Team
(Ariadna study 09/1301, 09/1301 CCN, PECS project Relativistic GNSS)



perturbed space-time

Earth multipoles
ocean tides, solid tides
Moon, Sun, Jupiter, Venus
relativity, Kerr

$$\vec{a} = \vec{a}_{GM} + \vec{a}_{2-6} + \vec{a}_{planets} + \vec{a}_{tides} + \vec{a}_{relativity} + \vec{a}_{Kerr}$$

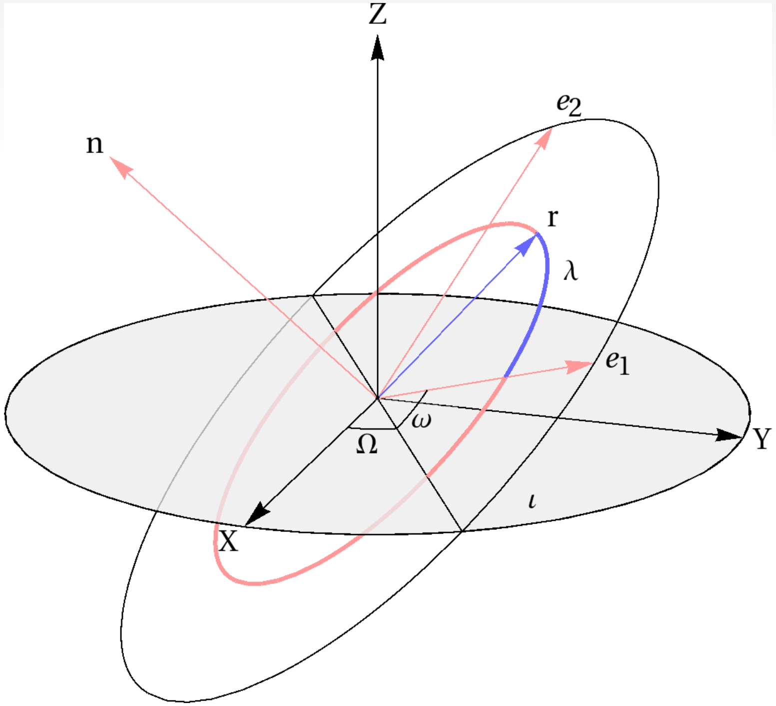
$$g_{\mu\nu} = \underset{\substack{\downarrow \\ \text{Schwarzschild}}}{g^{(0)}_{\mu\nu}} + \underbrace{h^{(2-6)}_{\mu\nu} + h^{(planets)}_{\mu\nu} + h^{(tides)}_{\mu\nu}}_{\text{Regge-Wheeler-Zerilli multipole expansion}} + h^{(Kerr)}_{\mu\nu}$$

Unperturbed orbits

$$H = \frac{1}{2} g^{(0)\mu\nu} p_\mu^{(0)} p_\nu^{(0)}$$

Orbital parameters = constants of motion

$$Q^i, P_i: \quad a, \epsilon, \omega, \Omega, \iota, t_a$$



$$\left.\begin{aligned} t &= t(\lambda | Q^i, P_i) \\ r &= r(\lambda | Q^i, P_i) \\ \theta &= \theta(\lambda | Q^i, P_i) \\ \phi &= \phi(\lambda | Q^i, P_i) \end{aligned}\right\} \text{analytical}$$

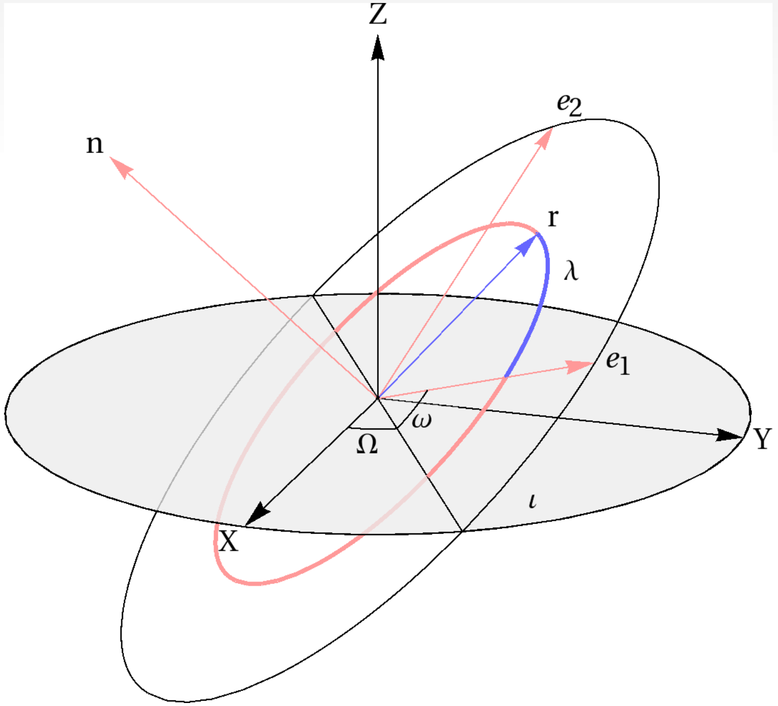
Perturbed orbits

$$H = \frac{1}{2} g^{(0)\mu\nu} p_\mu p_\nu - \underbrace{\frac{1}{2} h^{\mu\nu} p_\mu p_\nu}_{\Delta H}$$

Orbital parameters ≠ constants of motion

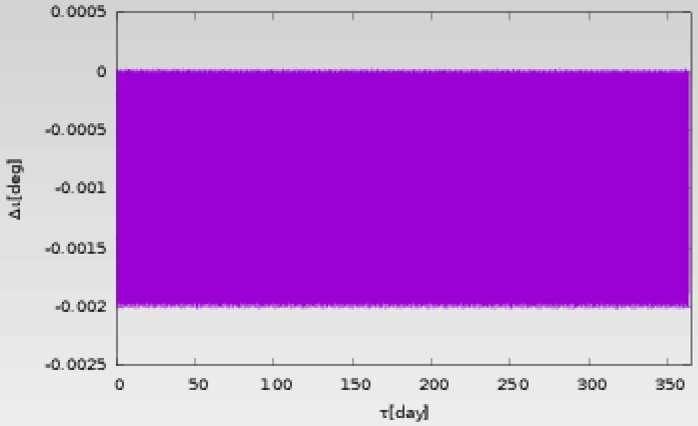
$Q^i, P_i: \quad a, \epsilon, \omega, \Omega, \iota, t_a$

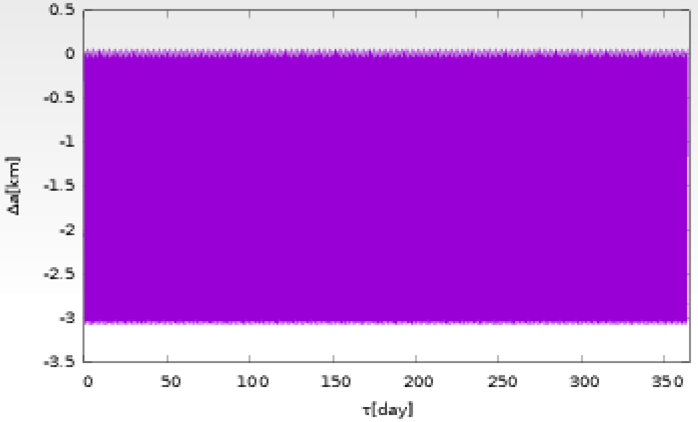
$$\dot{Q}^i = \frac{\partial \Delta H}{\partial P_i} \qquad \dot{P}_i = -\frac{\partial \Delta H}{\partial Q^i}$$

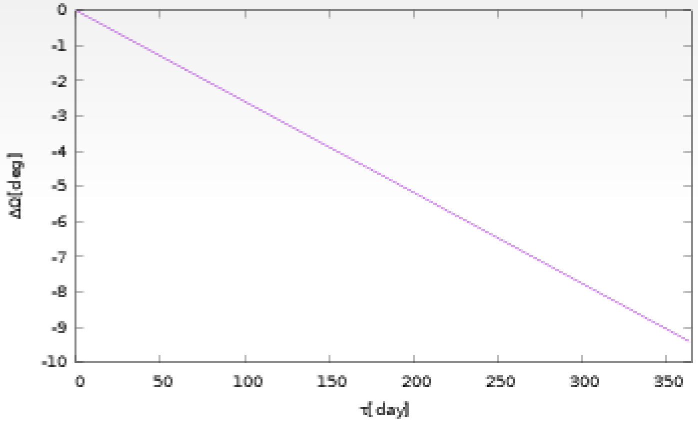


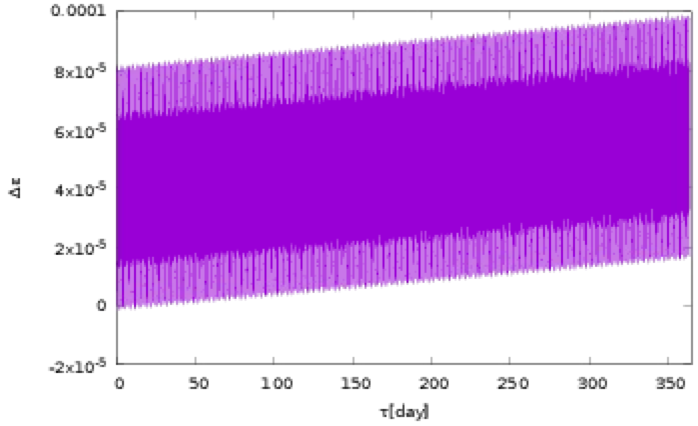
$$t = t(\lambda | Q^i(\tau), P_i(\tau))$$
$$r = r(\lambda | Q^i(\tau), P_i(\tau))$$
$$\theta = \theta(\lambda | Q^i(\tau), P_i(\tau))$$
$$\phi = \phi(\lambda | Q^i(\tau), P_i(\tau))$$

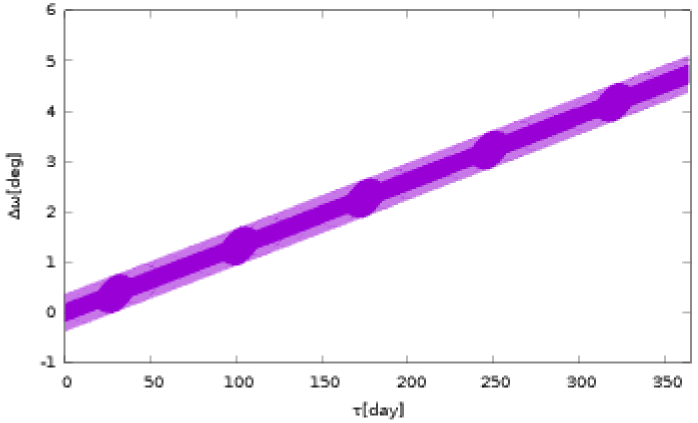
Earth multipoles

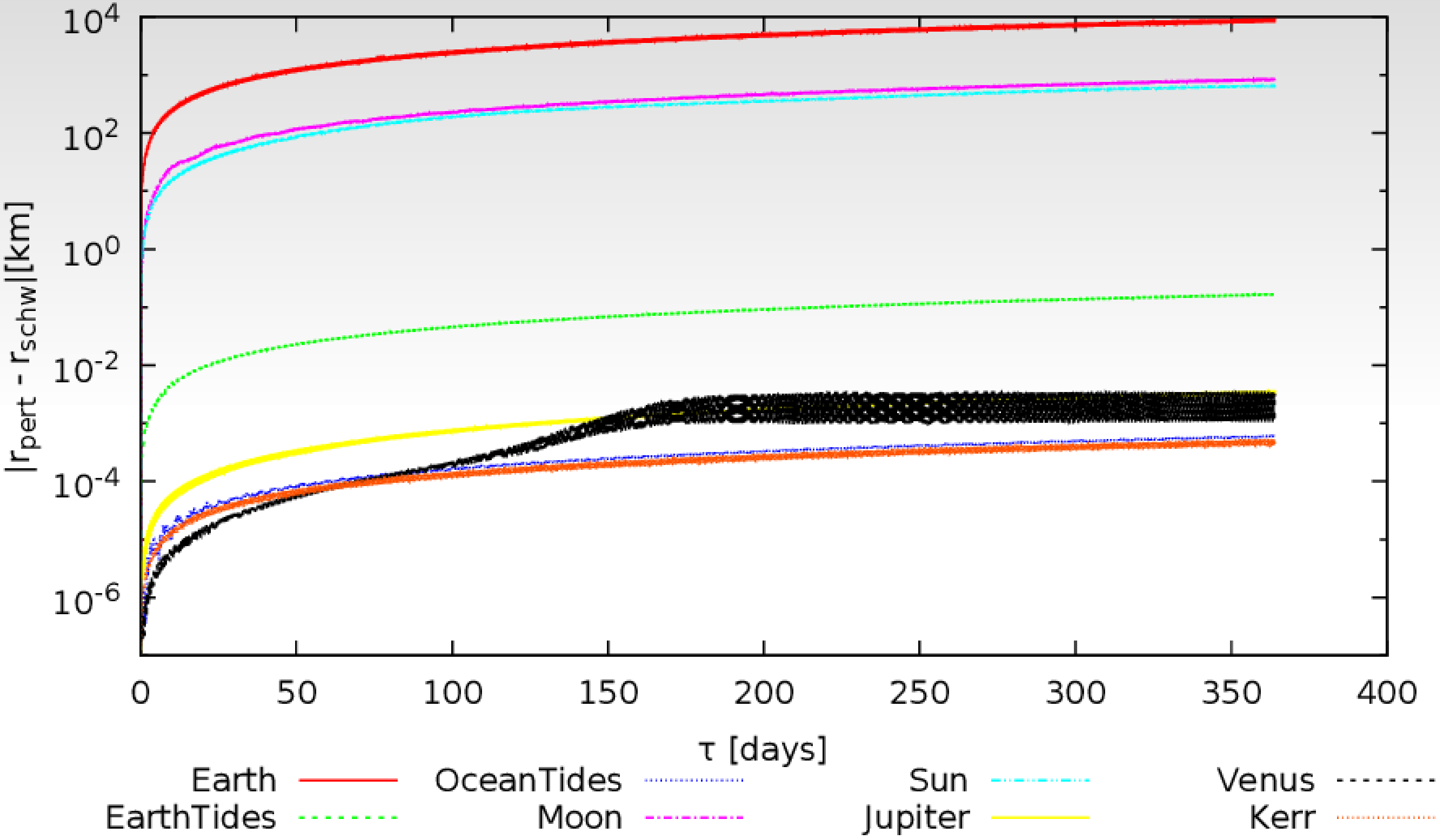




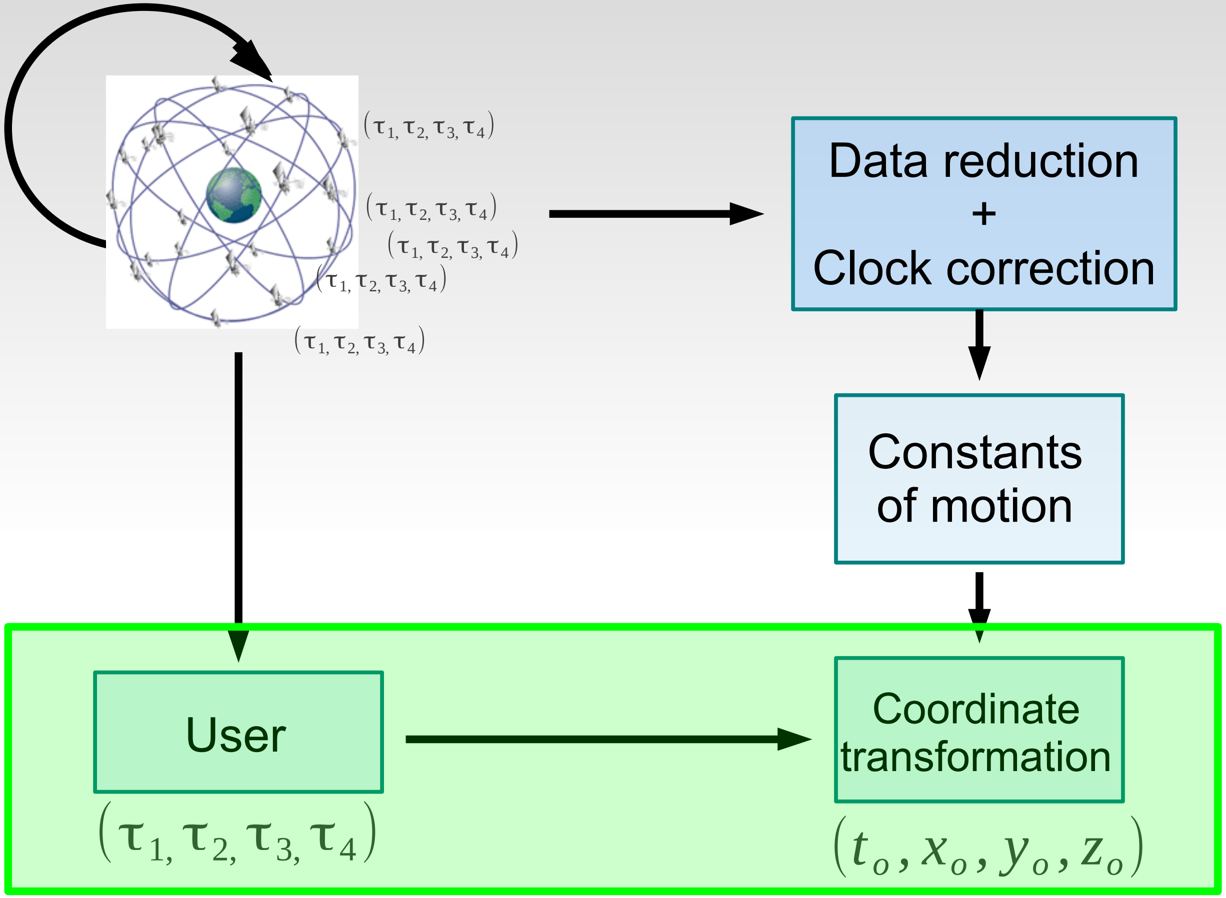







$|r_{pert} - r_{schw}|$[km]
10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
0
50
100
150
200
250
300
350
400
τ [days]
Earth
EarthTides
OceanTides
Moon
Sun
Jupiter
Venus
Kerr

Positioning algorithm in GR



$(\tau_1, \tau_2, \tau_3, \tau_4)$

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Data reduction
+
Clock correction

Constants
of motion

User

$(\tau_1, \tau_2, \tau_3, \tau_4)$

Coordinate
transformation

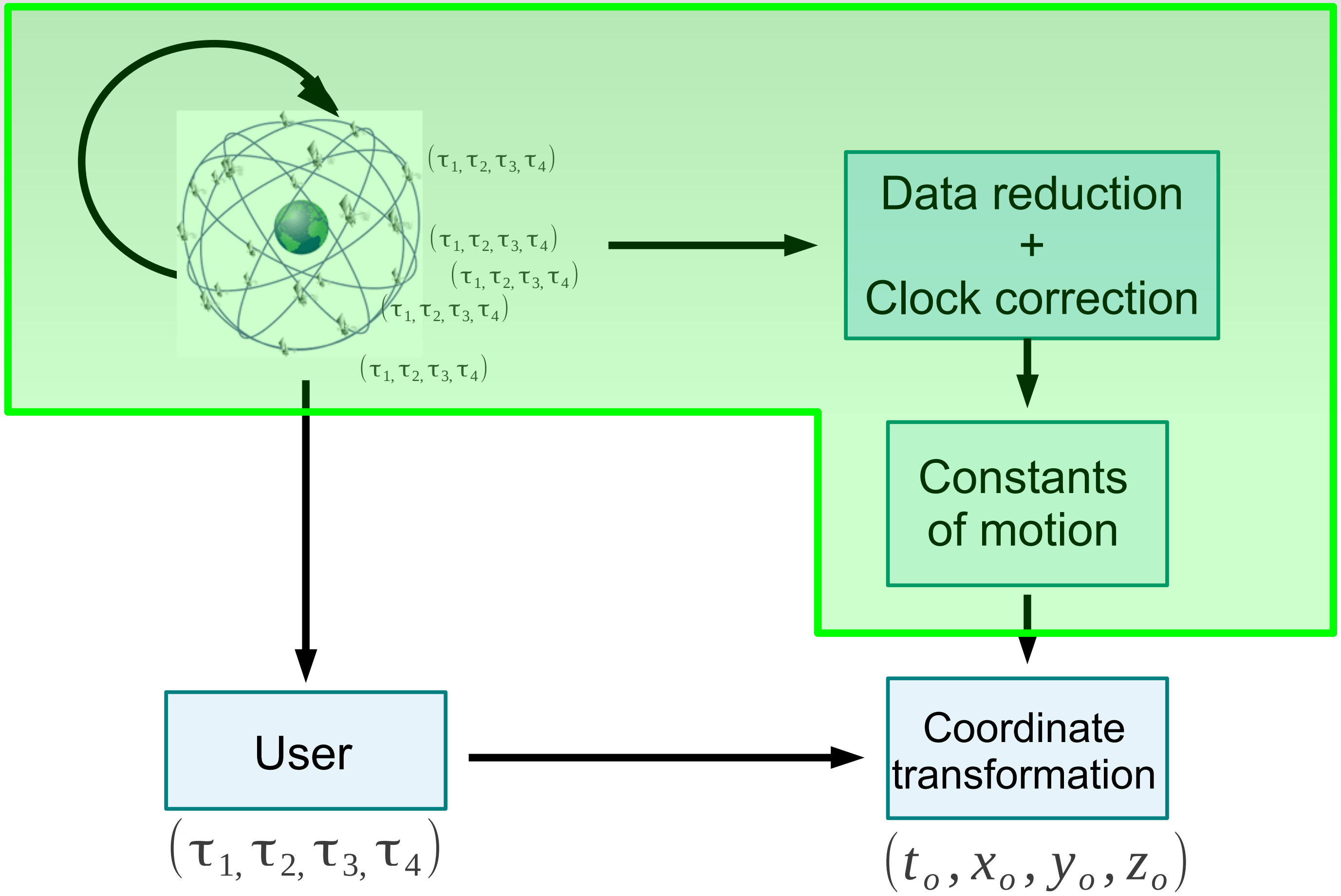
(t_o, x_o, y_o, z_o)

$\epsilon_t \sim 10^{-31}$

$\epsilon_{x,y,z} \sim 10^{-25}$

$T = 0.04\, s$

ABC – Autonomous Basis of Coordinates



$(\tau_1, \tau_2, \tau_3, \tau_4)$

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Data reduction
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Constants of motion

User

$(\tau_1, \tau_2, \tau_3, \tau_4)$

Coordinate transformation

(t_o, x_o, y_o, z_o)

ABC – Autonomous Basis of Coordinates

Title:2D_1.fig
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CreationDate:Sun Dec 1 20:02:37 2013

$$T_f = t_2(\bar{\tau}) - t_1(\tau)$$

ABC – Autonomous Basis of Coordinates

Title:2D_2nfig
Creator:fig2dev Version 3.2 Patchlevel 5
CreationDate:Sun Dec 1 20:09:46 2013

$$S\left(Q^i(0),P_i(0)\right)=$$

$$\sum_k \left\{ S_f\left(Q^i[k],P_i[k] \middle| Q^i\left(\tau[k]\right),P_i\left(\tau[k]\right)\right) \right.$$

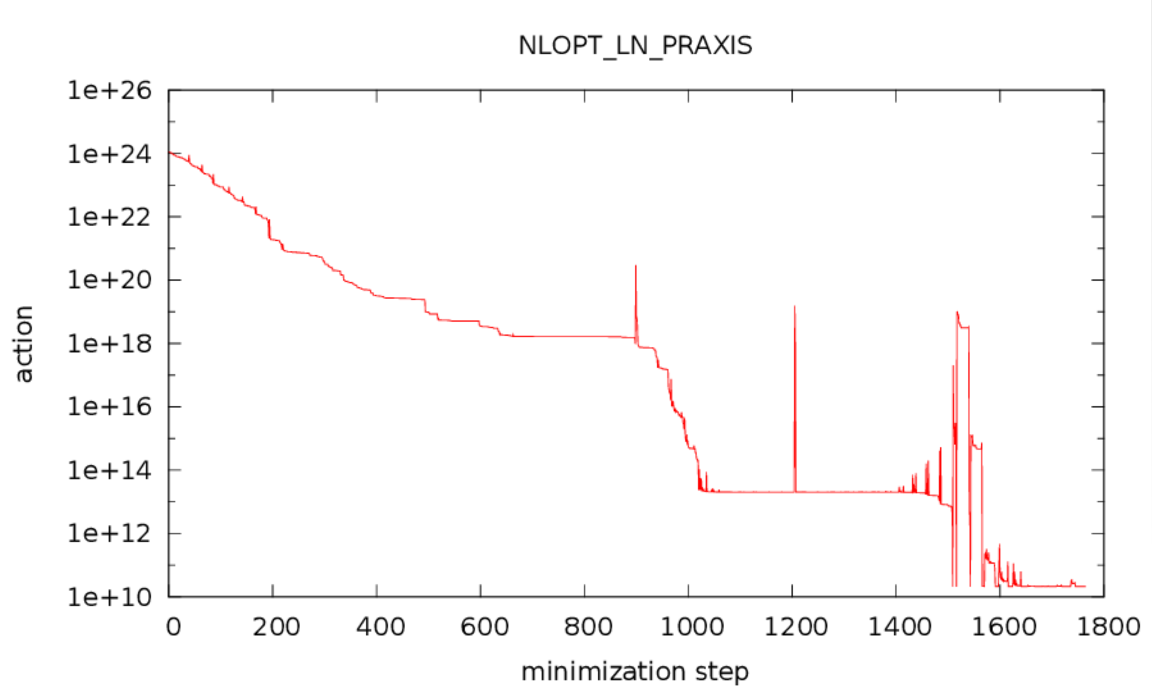
$$\left. -t_2\left(\bar{\tau}[k] \middle| Q^i\left(\bar{\tau}[k]\right),P_i\left(\bar{\tau}[k]\right)\right)+t_1\left(\tau[k] \middle| Q^i\left(\tau[k]\right),P_i\left(\tau[k]\right)\right)\right\}^2$$

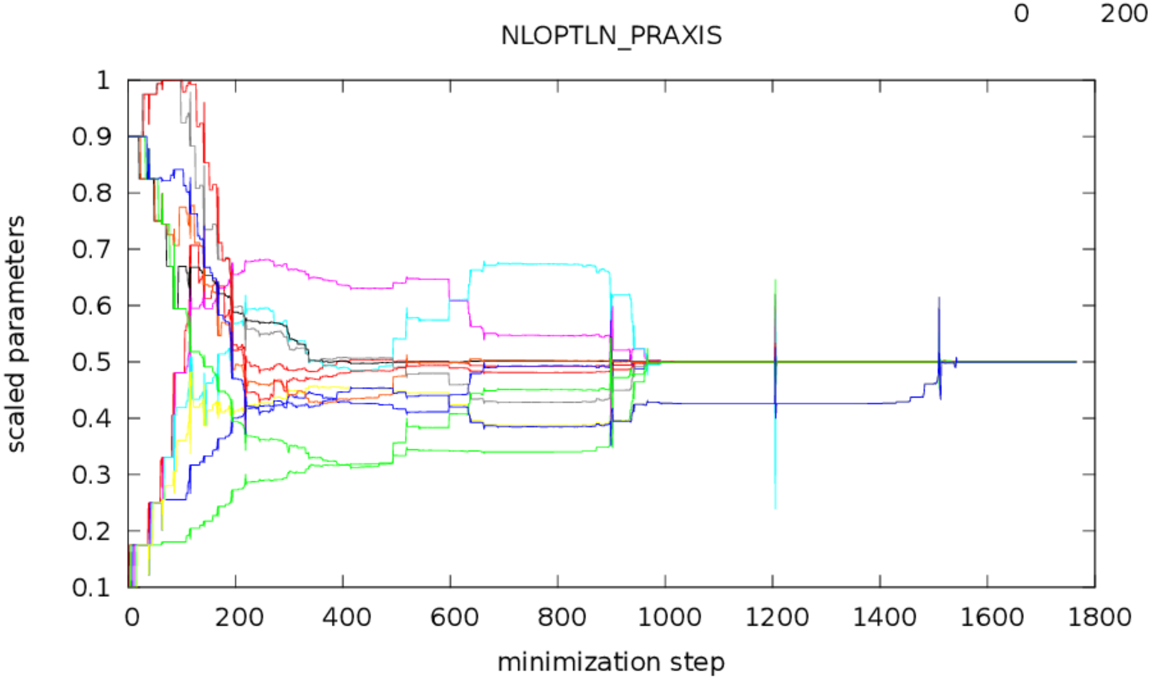
$$T\left(Q^i(0),P_i(0)\right)=\left\{T\left(\tau,\bar{\tau} \middle| Q^i(\tau),P_i(\tau)\right)\right.$$

$$\left.-t_2\left(\bar{\tau} \middle| Q^i(\bar{\tau}),P_i(\bar{\tau})\right)+t_1\left(\tau \middle| Q^i(\tau),P_i(\tau)\right)\right\}^2$$

ABC – Autonomous Basis of Coordinates

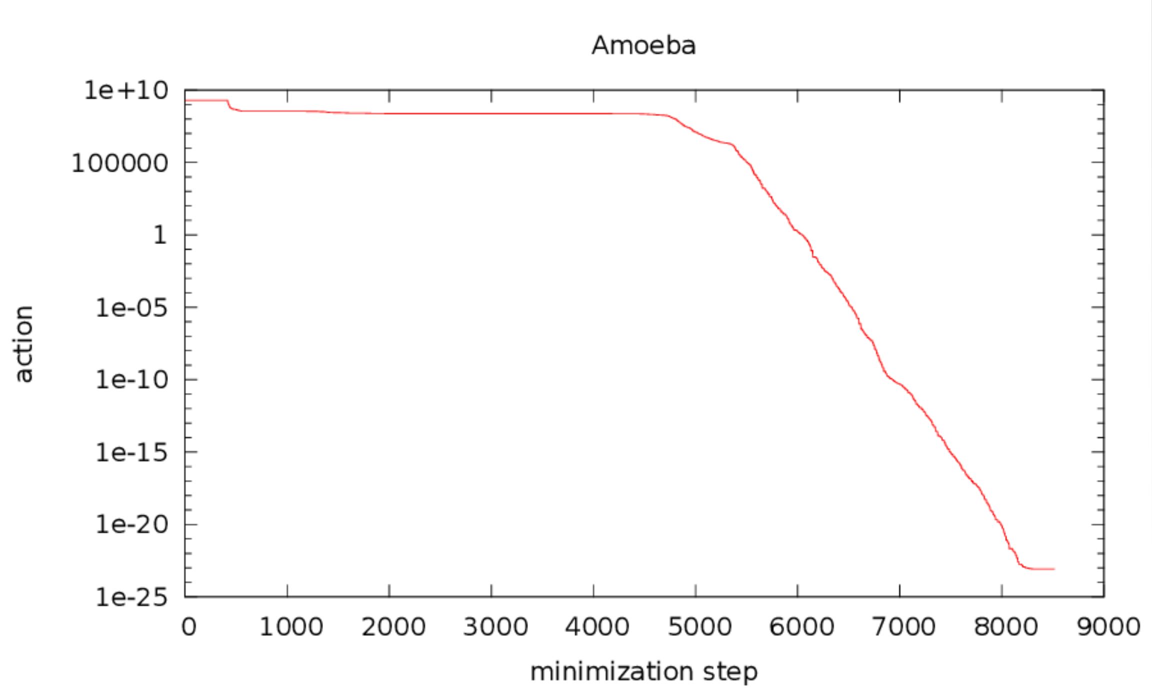
12D minimization

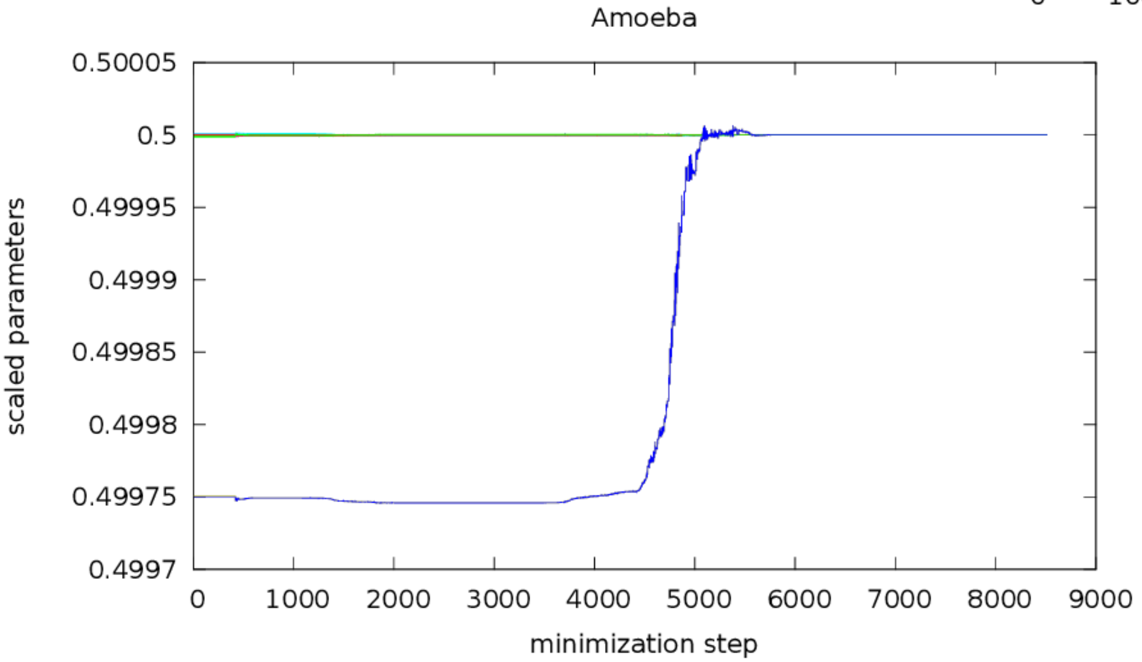




ABC – Autonomous Basis of Coordinates

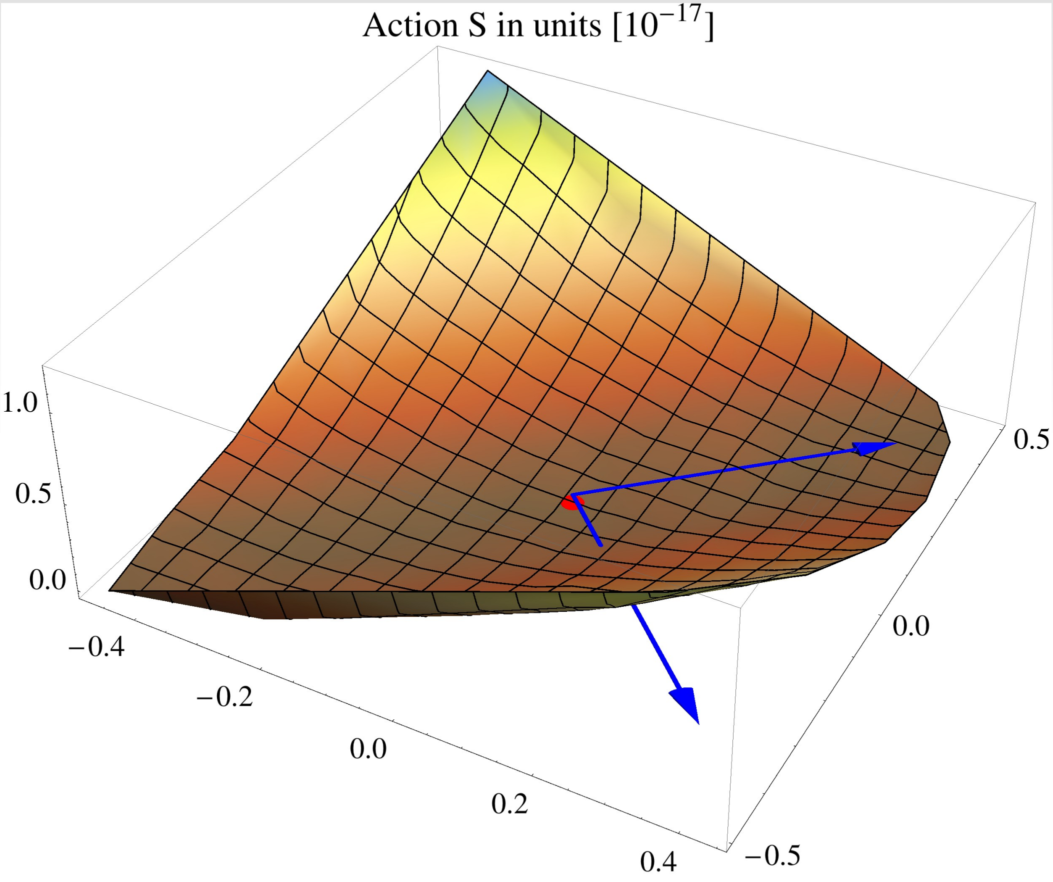
12D minimization





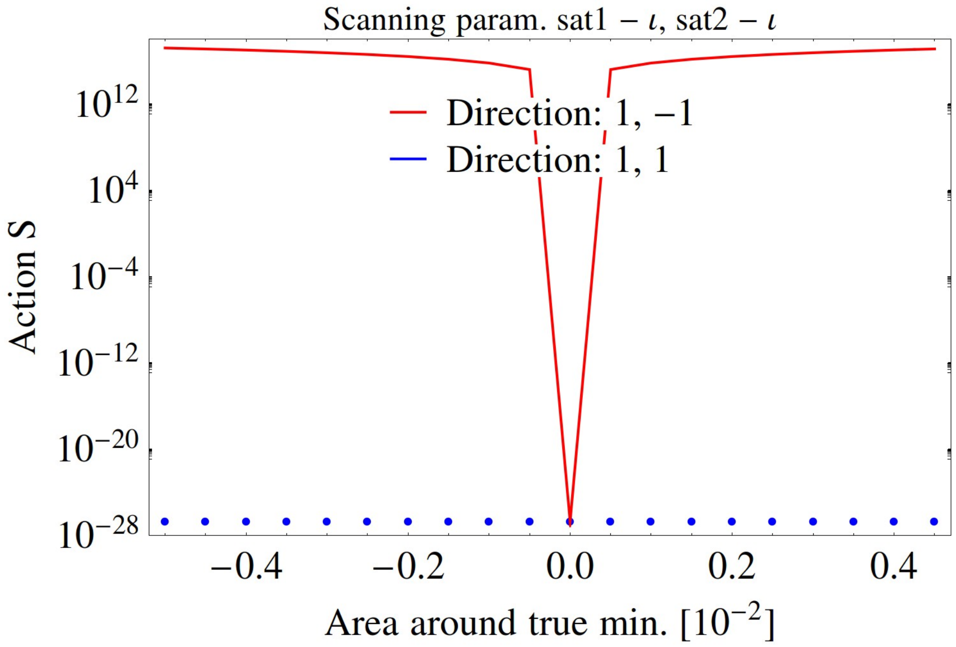
$$\epsilon\left(Q^i(0), P_i(0)\right) \sim 10^{-22}$$

ABC – Degeneracies

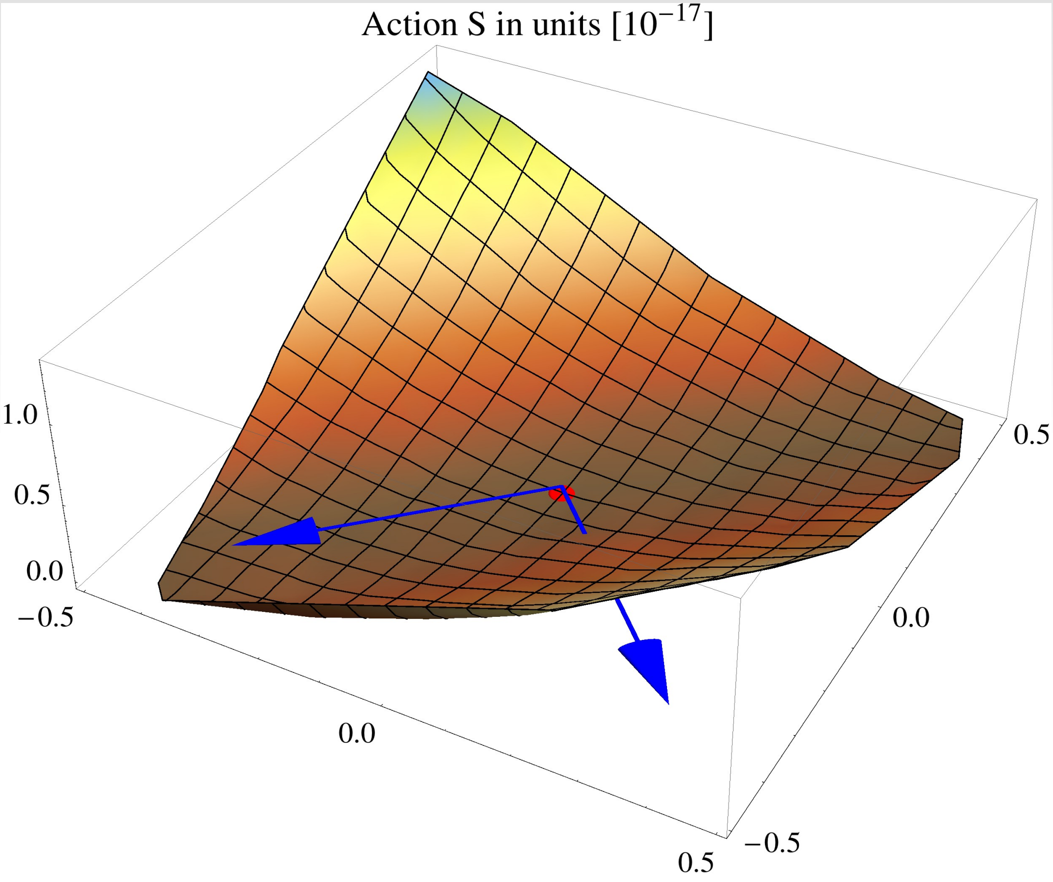


no perturbations

$\Delta\iota = \pm 0.01$

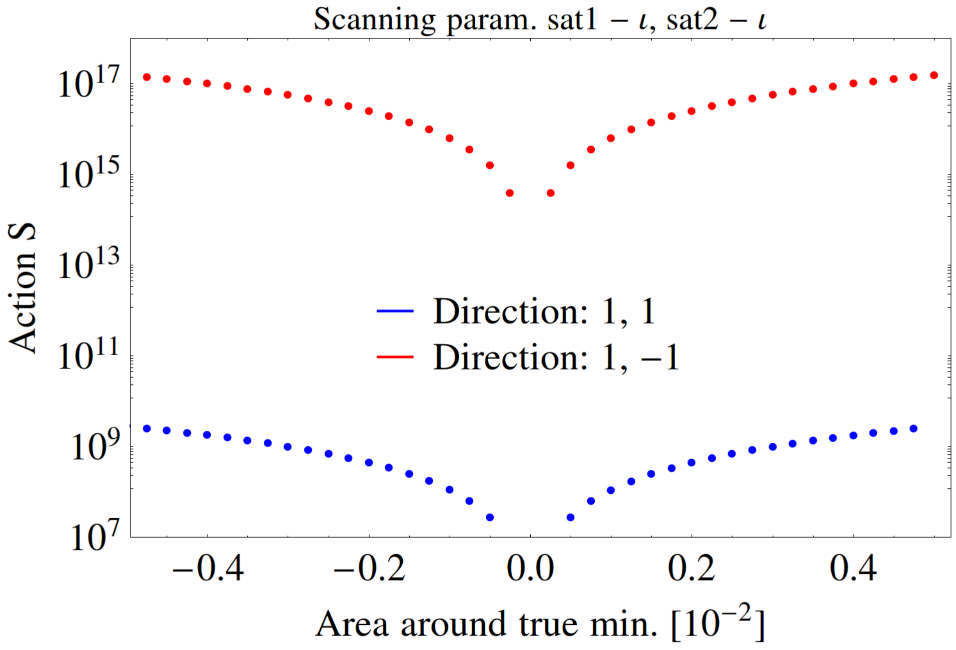


ABC – Degeneracies

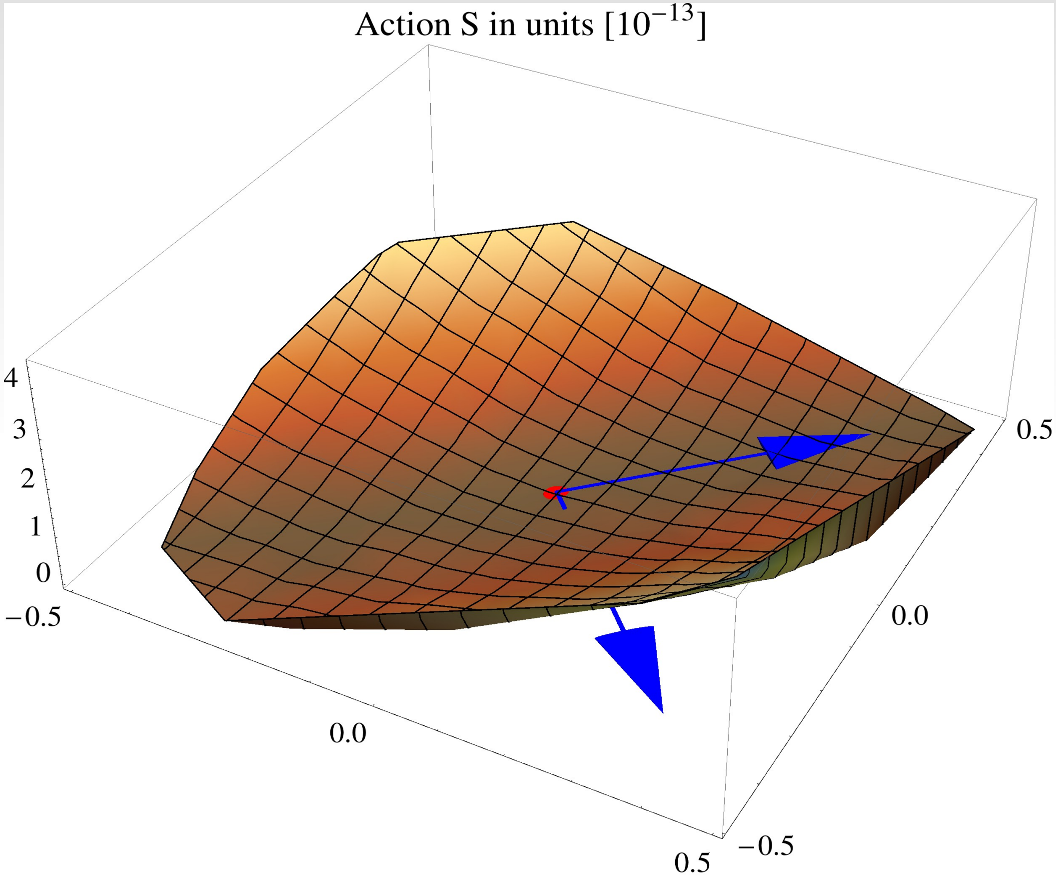


Earth multipoles
Solid tides
Ocean tides

$\Delta\iota = \pm 0.01$

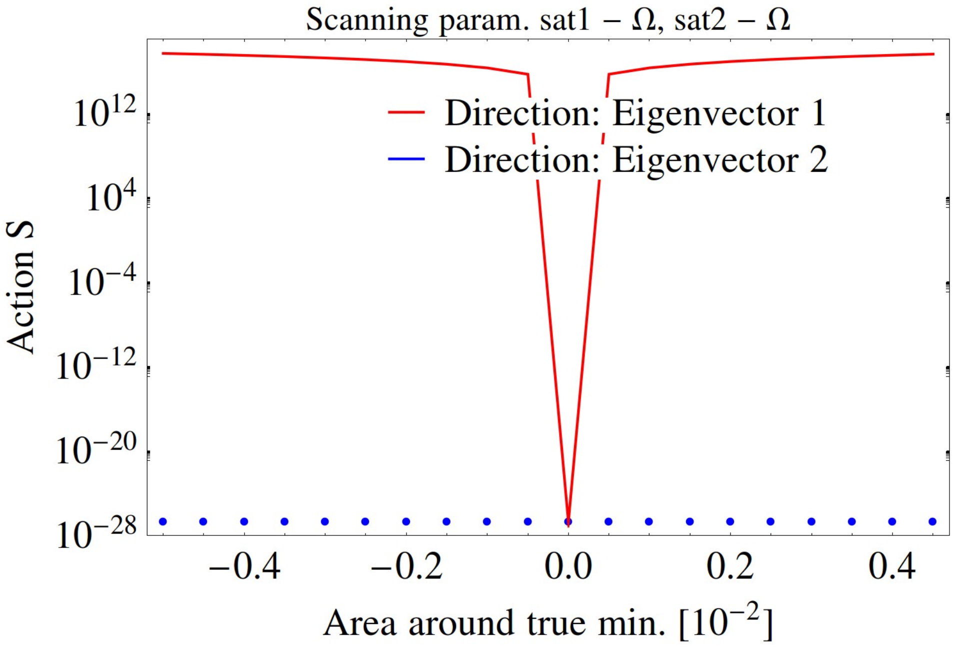


ABC – Degeneracies

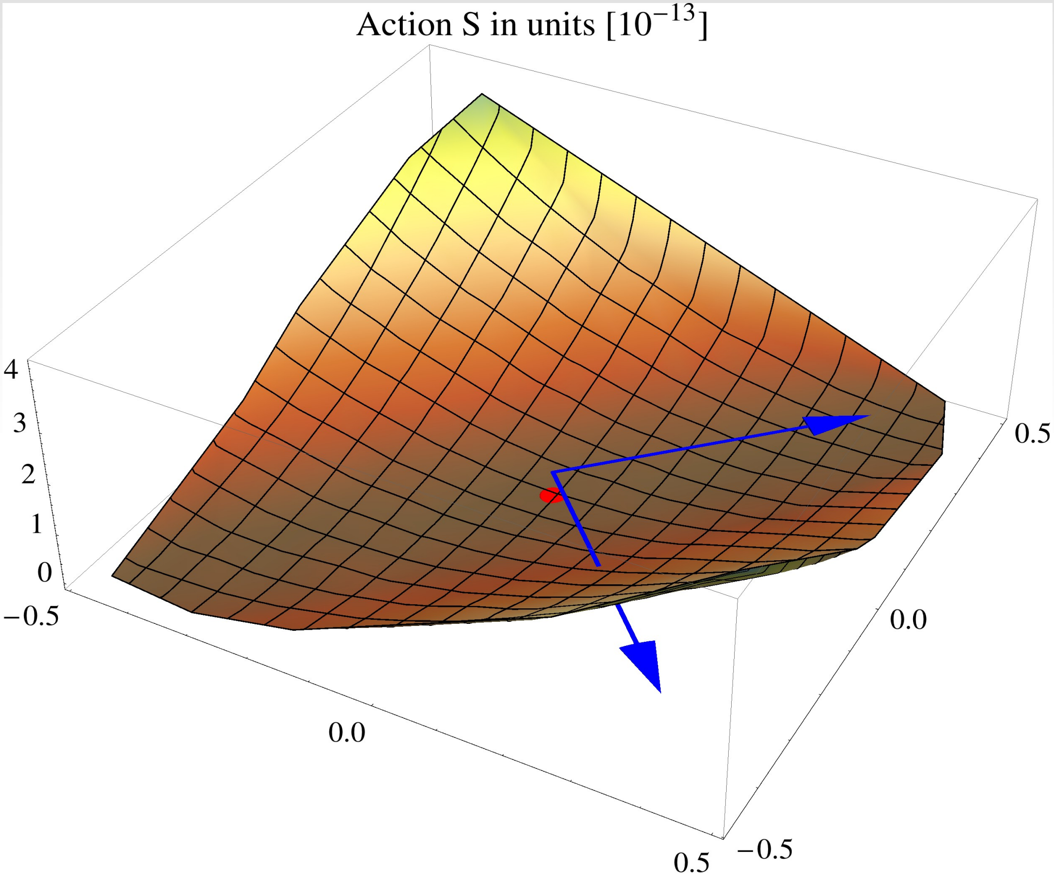


no perturbations

$\Delta\Omega = \pm 2 \times 10^{-4}$

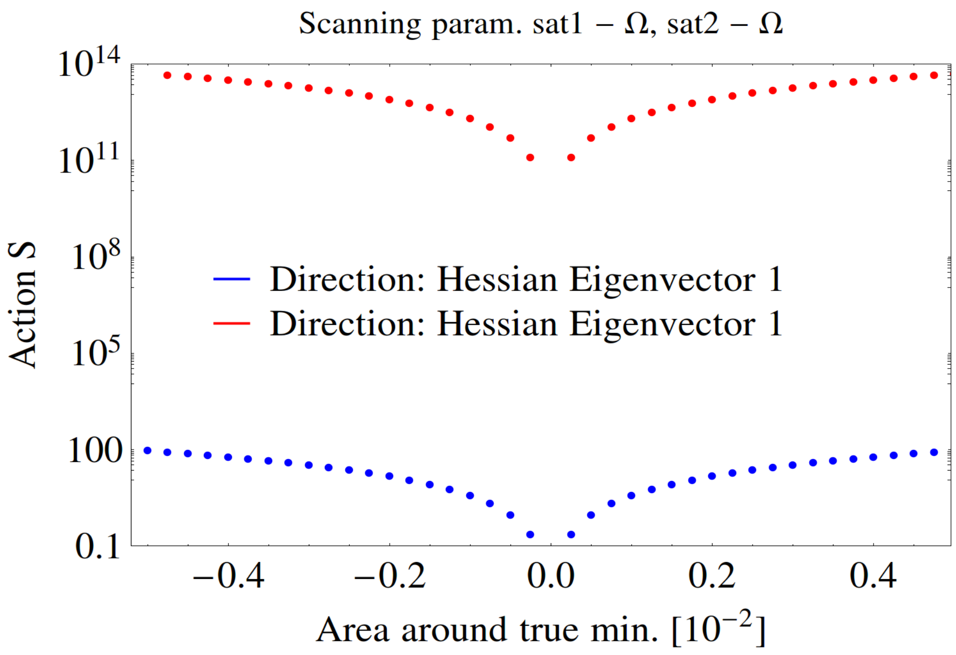


ABC – Degeneracies

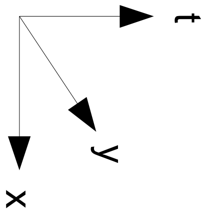


Earth multipoles
Solid tides
Ocean tides

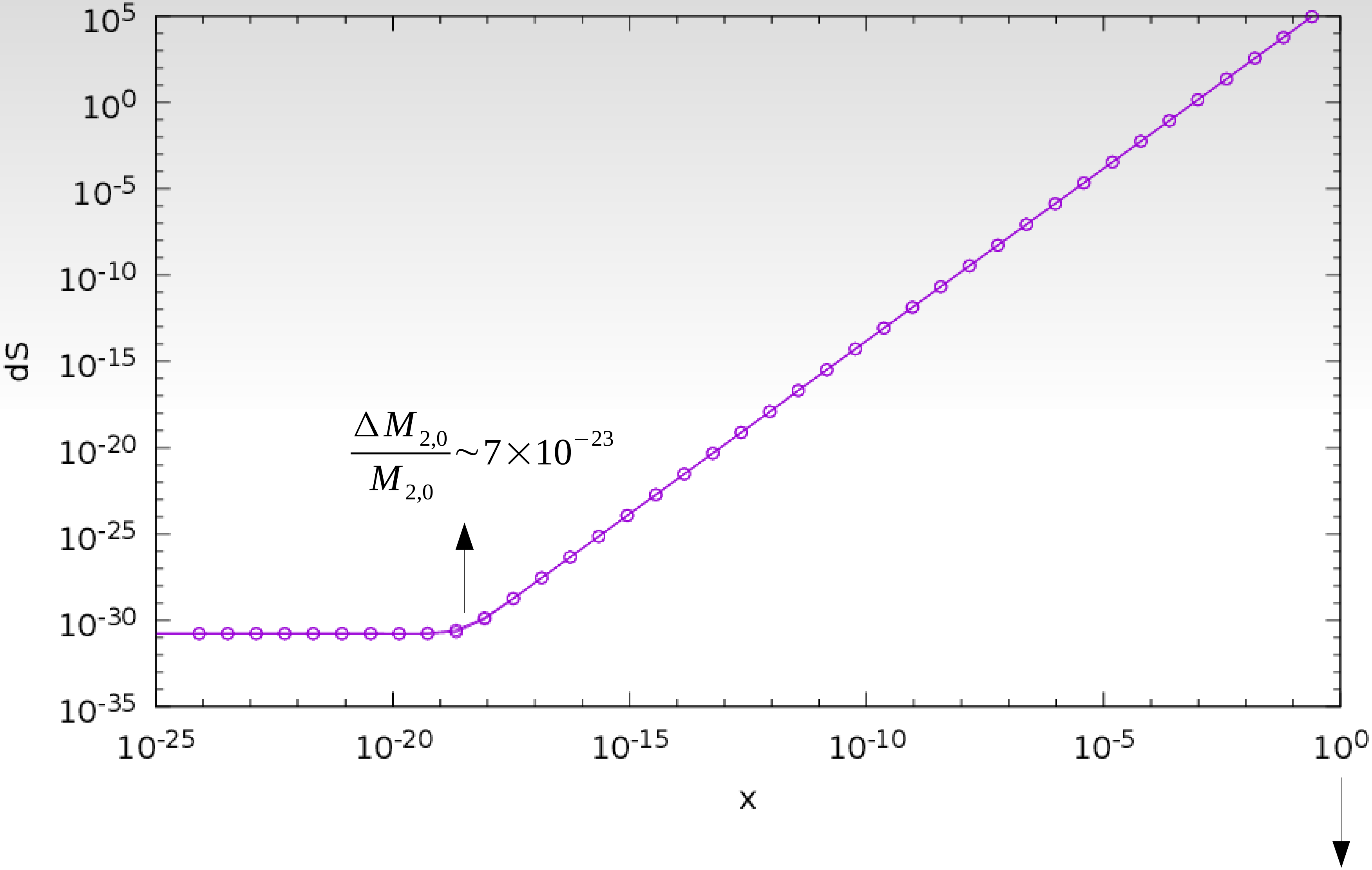
$\Delta\Omega = \pm 2 \times 10^{-4}$



ABC – Autonomous Basis of Coordinates



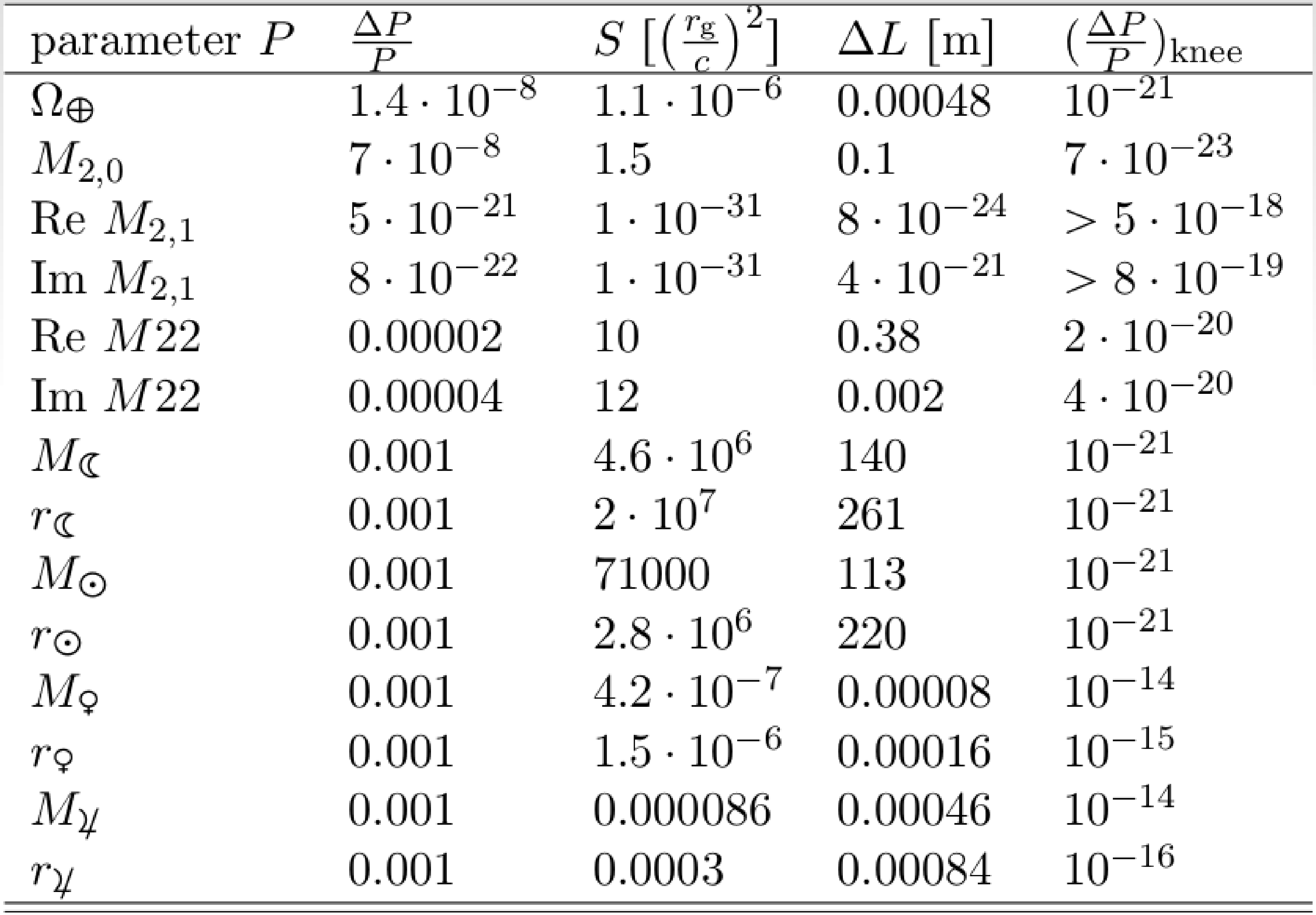
ABC – Refinement of gravitational parameters



$$\frac{\Delta M_{2,0}}{M_{2,0}} \sim 7\times10^{-23}$$

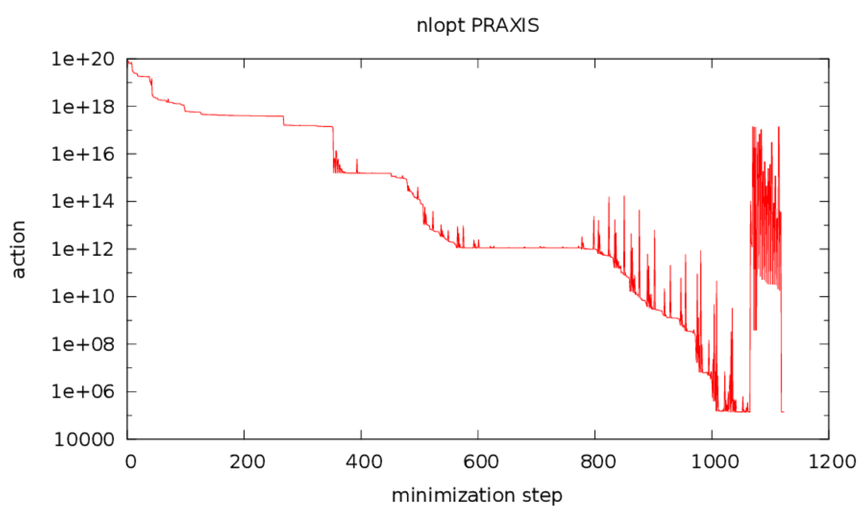
$$\frac{\Delta M_{2,0}}{M_{2,0}} = 7\times10^{-5}$$

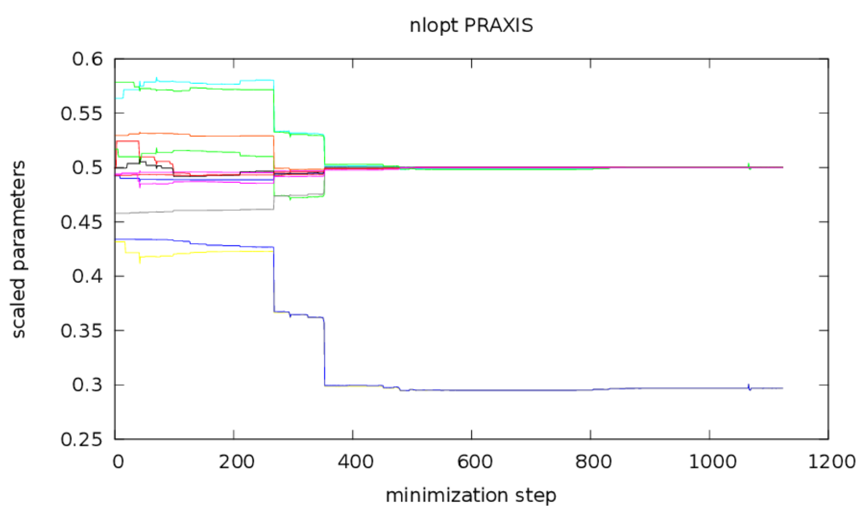
ABC – Refinement of gravitational parameters

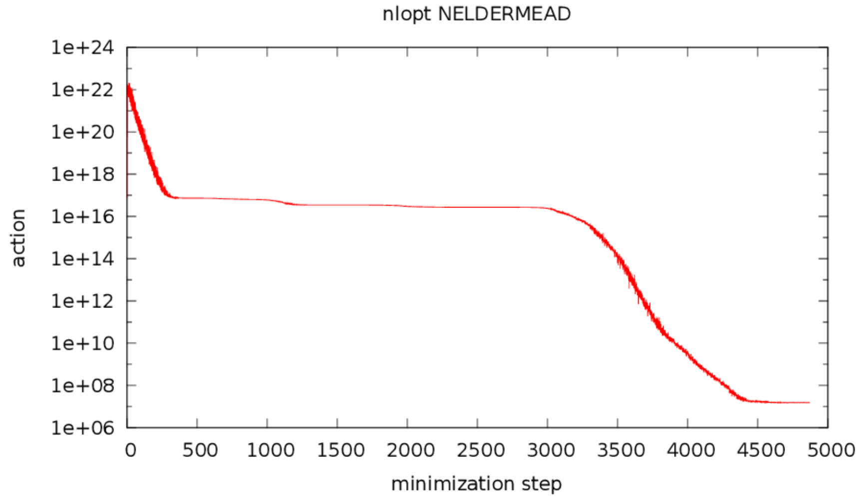


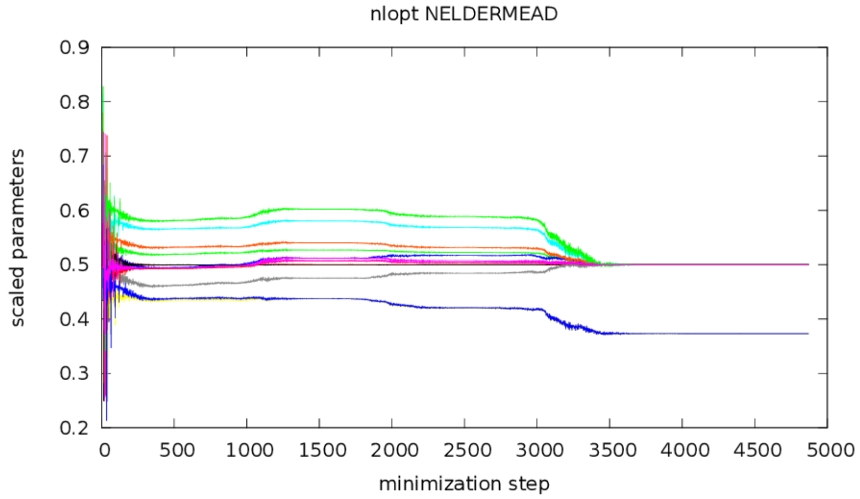
parameter P	$\frac{\Delta P}{P}$	$S\ [(\frac{r_{\mathrm{g}}}{c})^2]$	ΔL [m]	$(\frac{\Delta P}{P})_{\mathrm{knee}}$
$\Omega_\oplus$	$1.4 \cdot 10^{-8}$	$1.1 \cdot 10^{-6}$	0.00048	10^{-21}
$M_{2,0}$	$7 \cdot 10^{-8}$	1.5	0.1	$7 \cdot 10^{-23}$
Re $M_{2,1}$	$5 \cdot 10^{-21}$	$1 \cdot 10^{-31}$	$8 \cdot 10^{-24}$	$> 5 \cdot 10^{-18}$
Im $M_{2,1}$	$8 \cdot 10^{-22}$	$1 \cdot 10^{-31}$	$4 \cdot 10^{-21}$	$> 8 \cdot 10^{-19}$
Re $M22$	0.00002	10	0.38	$2 \cdot 10^{-20}$
Im $M22$	0.00004	12	0.002	$4 \cdot 10^{-20}$
$M_{\leftmoon}$	0.001	$4.6 \cdot 10^{6}$	140	10^{-21}
$r_{\leftmoon}$	0.001	$2 \cdot 10^{7}$	261	10^{-21}
$M_\odot$	0.001	71000	113	10^{-21}
$r_\odot$	0.001	$2.8 \cdot 10^{6}$	220	10^{-21}
$M_{\venus}$	0.001	$4.2 \cdot 10^{-7}$	0.00008	10^{-14}
$r_{\venus}$	0.001	$1.5 \cdot 10^{-6}$	0.00016	10^{-15}
$M_{\jupiter}$	0.001	0.000086	0.00046	10^{-14}
$r_{\jupiter}$	0.001	0.0003	0.00084	10^{-16}

ABC – Refinement of gravitational parameters









ABC – Autonomous Basis of Coordinates

- **Robustness** of recovering constants of motion with respect to noise in the data
- **Consistency** of description with redundant number of satellites
- Possibility to use the constellation as a **clock with long term stability**

- Its realization does not rely on observations from Earth
  - No entanglement with Earth internal dynamics
  - No Earth stations for maintaining of the frame
- Stability and accuracy
  - Based on well-known satellite dynamics
  - Satellite orbits are very stable in time, and can be accurately described
- Applications in science
  - geophysics, relativistic gravitation and reference frames
  - determine/refine values of gravitational parameters (e.g. multipoles)

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CreationDate:(Sunday, D