05/4106 Spiral Trajectories in Global Optimisation of Interplanetary and Orbital Transfers

Type of activity: Medium Study (4 months, 25 KEUR)

Background

In a Euclidean space dominated by the gravity of one single massive body, the trajectories of bodies are conic sections. The problem of "choosing" one of such trajectories to transfer a body from one planet to another, maybe exploiting also the gravitational kicks (swing-bys) that close passages to a number of intermediate planets may offer, may be considered and studied as a global optimisation problem[1-2]. The objective function may be defined in a number of different ways according to the particular problem treated. It has been recently shown[1] that the complexity of such an optimisation, named Multiple Gravity Assist (MGA) optimisation, is quartic in the number of swing-bys and cubic in the time resolution chosen whenever a space pruning technique, specifically designed for the problem, is implemented. The technology developments in space propulsion, and in particular the electric propulsion systems, made it meaningful to space mission designer to start considering non-conic solutions to the planetary transfer problem. Some analytical solutions have already been found: Pinkham Spirals[5], Ward Spirals[3-4] Exponential Sinusoids[6], Keplerian Low-Thrusts[12] and potential Wells[8] are only some of the most important. It is actually possible to build an arbitrary large number of analytical solutions by an inverse dynamical approach as shown by Petropoulos[9] and exploited more recently by Patel[11]. De Pascale et al.[10], during their work in the Advanced Concepts Team, proposed a shape-based approach to solve the three dimensional case at the cost of violating the dynamic in a least square sense.

Among all these special solutions, though, exponential sinusoids are, maybe, the most surprising and represent more than an efficient numerical approach aiming at finding a low-thrust solution. The definition of the polar shape of the trajectory, and not of the whole state, together with a tangential thrust hypothesis, allows determining some elegant expressions for the thrust magnitude and the velocity vector. The practical use of these solutions proposed in the original paper by Petropoulos and Longuski is not suitable to be incorporated into a formalization of the MGA problem as a global optimisation problem having as search space the departure, fly-by and arrival dates. A different theoretical approach to exponential sinusoids[7] allows for this kind of formalization and enables the possibility of using the results and techniques derived for the simpler ballistic MGA. The enabling theoretical result is the solution of Lambert's problem for the exponential sinusoids, a result that has yet to be embedded into a design tool to verify its ability to locate useful first guesses trajectories. The extreme simplicity and computational efficiency of this approach could compensate for the known shortcomings of the exponential sinusoids (boundary conditions are violated).

Study Objectives

The objective of this study is to embed the Lambert's problem solution for exponential sinusoids into a global optimisation algorithm in order to produce fast and efficient first guess trajectories for the interplanetary trajectory problem. The study should also determine to what extent the use of these special solutions in connection with global optimisation techniques is able to locate trajectories close to a real optimum.

In summary the study objectives are as follows:

- Implement the solution of the Lambert's problem for exponential sinusoids[7] in a global optimisation scheme, possibly Differential Evolution, using the launch, fly-by and arrival dates as variables, and study the problem complexity.

- Expand the results obtained by pruning the search space[1] to the case in which exponential sinusoids are allowed in addition to ballistic arcs to connect the planets.

- Assess the results achievable in the optimisation of rendezvous trajectories with asteroids and comets and the optimality of the solutions.

Proposals and assessments on different ideas related to the exploitation of spiral trajectories are also welcomed.

The study will be performed in close cooperation with the ACT, and its results should be at a high standard that makes them suitable for publication in a peer reviewed journal.

References

1. Myatt, D.R., Becerra, V.M., Nasuto, S.J., Bishop, J.M.: "Advanced Global Optimisation for Mission Analysis and Design", ESA ARIADNA final report AO4532/18138/04/NL/MV, also available on-line at www.esa.int/gsp/ACT/studies_publications.htm

2. Di Lizia, P., Radice, G.: "Advanced Global Optimisation for Mission Analysis and Design", ESA ARIADNA final report AO4532/18139/04/NL/MV, also available on-line at www.esa.int/gsp/ACT/studies_publications.htm

3. Rimrott, F.P.J. and Salustri, F.A., "On the Ward Spiral differential equation", CSME Transactions, 28(1):1-6, 2004.

4. Rimrott. F.P.J. and Salustri, F.A., "Orbit dynamics and Ward Spirals"; Paper IAF-01-A.6.09, 52nd Intl Astronautical Congress, Toulouse, 2001.

5. Pinkham, G. "Reference Solution for Low Thrust Trajectories", J. American Rocket Society, Vol.32, No.5, pp.775 76, May 1962.\

6. Petropoulos, A.E. and Longuski, J.M., "Shape-Based Algorithm for Automated Design of Low-Thrust, Gravity-Assist Trajectories", Journal of Spacecrafts and Rockets, Vol. 41, No. 5, 2004

7. Izzo, D., “Lambert’s problem for exponential sinusoids”, ACT/ESTEC internal report, available on-line http://www.esa.int/gsp/ACT/doc/ACT-RPT-DI-4100-LMSP01.pdf, April 2005.

8. Prussing, J.E. and Coverstone, V., “Constant Radial Thrust Acceleration Redux”, Journal of Guidance Control and Dynamic Vol. 21, No. 3, 1998.

9. Petropoulos, A.E., “Some Analytic Integrals of the Averaged Variational Equations for a Thrusting Spacecraft”, IPN Progress Report 42-150 August 15, 2002.

10. De Pascale, P., Vasile M., Casotto, S., “Preliminary Analysis of Low-Thrust Gravity Assist Trajectories by an Inverse Method and a Global Optimisation Technique” 18th International Symposium on Space Flight Dynamics, 11-15 October 2004, Munich, Germany.

11. Patel, P., Scheeres, D., Zurbuchen, T., “A Shape Based Approach to Spacecraft Trajectories: Analysis and Optimization”, Paper AAS 05-130

12. Markopoulos, N., “Non-Keplerian Manifestations of the Keplerian Trajectory Equation, and a Theory of Orbital Motion Under Continuous Thrust”, Paper AAS 95-217