Global Optimisation Competition Workshop

Presentation of DEIMOS Results

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Table of Contents

- The design of space trajectories leads to two type of mathematical optimisation problems:
  - Optimisation of a discrete number of mission parameters to minimise a cost function subject to equality and inequality constraints: **Non-linear Programming** (**NLP**) problem
  - Optimisation of the time evolution of a set of control variables to minimise a cost function subject to initial, final and path, equality and inequality constraints: **Constrained Optimal Control** (**OC**) problem

Mathematical Problem	Propulsion system or Trajectory	Trajectory Design Problem
Constrained Parameter Optimisation or **Non-Linear (NLP) Programming**	•Chemical propulsion •Impulsive manoeuvre •Heliocentric arcs •Powered swingbies •Unpowered swingbies	•Optimum orbit transfer •Optimum orbit control •Interplanetary design •Weak Stability Boundary Transfers
Constrained Optimal Control (OC) Problem	•Solar Electric propulsion •Ion engine •Low Atmosphere flight •Rocket engine	•Interplanetary Low Thrust trajectory •Atmospheric Re-entry •Aeroassisted transfer •Aerobraking •Launcher ascent

- The constrained parameter optimisation problem is formulated as follows:
  - Find the minimum of a cost function **f(x)**
  - Where **x** is a vector of parameters with dimension **n**
  - Subject to
    - Equality constraints: $g_i(x) = 0, i=1, ..., m_{eq}$
    - Inequality constraints: $g_i(x) > 0, i=m_{eq}+1,...,m$

Constrained Parameter Optimisation

- For equality constraints, the mathematical solution is obtained by introducing the Lagrange multipliers (1755), and the augmented cost function:

$$L(x,\lambda) = f(x) + \sum_{i=1}^{m_{eq}} \lambda_i g(x)_i$$

- The necessary optimality conditions are:

$$\frac{\partial L(x,\lambda)}{\partial x_i} = 0$$

$$\frac{\partial L(x,\lambda)}{\partial \lambda_i} = 0$$

- For inequality constraints, the mathematical problem is more complex. The solution is given by introducing the Karush – Kuhn – Tacker conditions:

$$\frac{\partial f(x)}{\partial x_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial g_j(x)}{\partial x_i} = 0, i = 1, ..., n$$

$$g_i(x) = 0, i = 1, ..., m_{eq}$$

$$g_i(x) \geq 0, i = m_{eq}, ..., m$$

$$g_i(x)\lambda_i \geq 0, i = m_{eq}, ..., m$$

$$\lambda_i \leq 0, i = m_{eq}, ..., m$$

$$Z^T(x)W(x,\lambda)Z(x) \geq 0$$

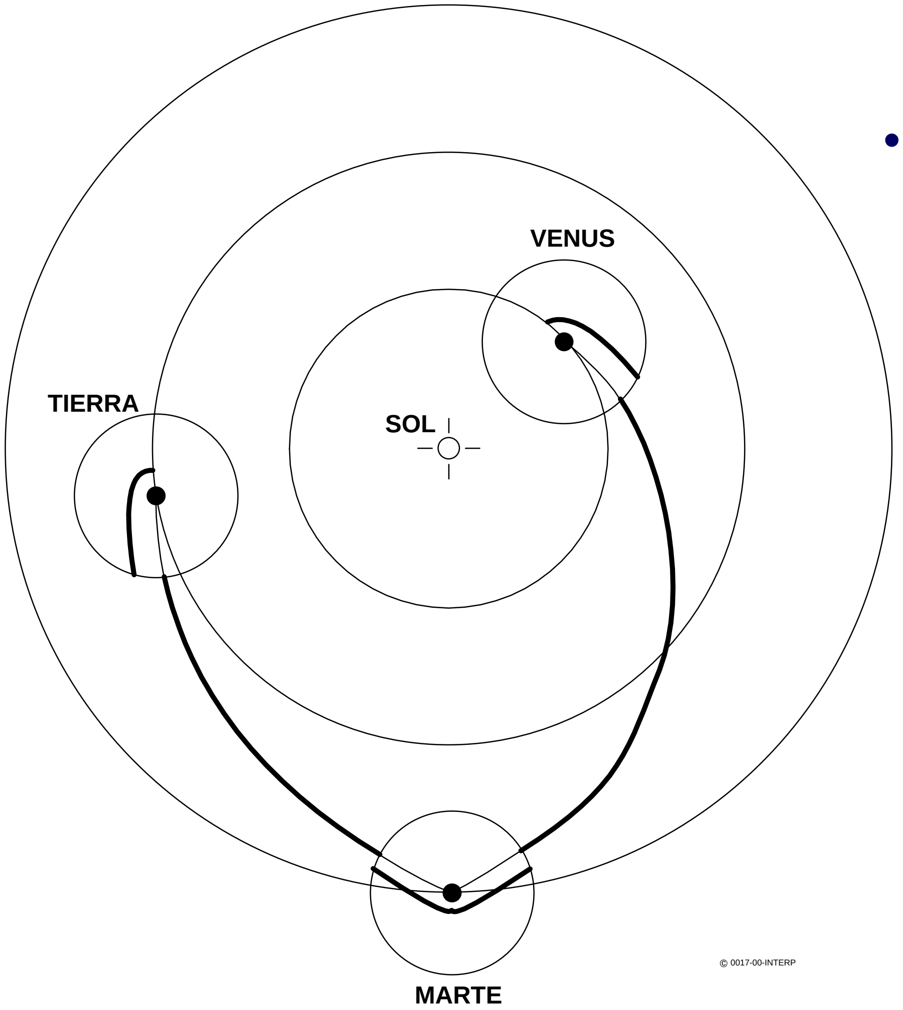
- Where W is the Hessian of the Lagrange function and the columns of the Z matrix are a base of orthogonal vectors to the Jacobian of the constraints

Constrained Parameter Optimisation

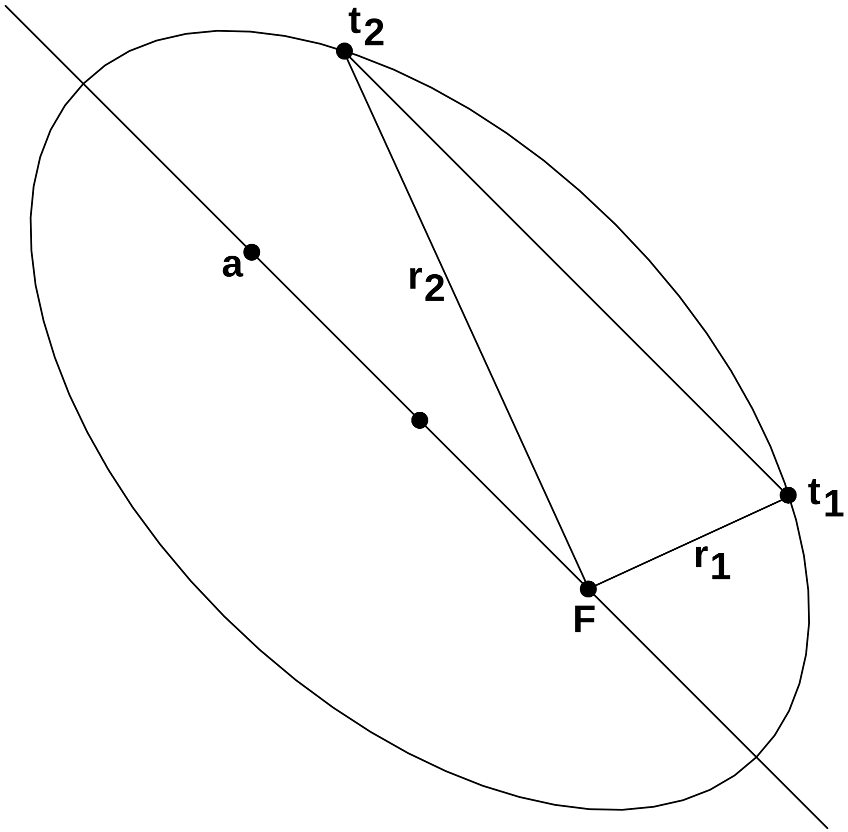
Step	Objective	Comments
1. Problem Formulation	• Identification of the most suitable set of parameters • Identification of equality and inequality constraints	•Scale factors for: –variables, –constraints and –cost function •Technical constraints
2.- Initial Guess of Solution (most difficult step)	•Find a first trajectory: –Systematic scan –Genetic algorithm –Simplified methods (patched conics)	•Gradient methods are very sensitive to the initial guess •To ensure global minimum, an exhaustive search must be done
3.- NLP Solver Method	Identify the most suitable NLP solver: deterministic, probabilistic or hybrid.	Gradient methods present good convergence but local minima can be found

Deterministic	**Search**	**Fibonacci, uniform, asymmetric**
	Gradient	**OPRQP: recursive quadratic algorithm with penalty function (Biggs, NOC, Hatfield)**
	Conjugated Directions	**Davidon Fletcher Powell, Fletcher & Reeves, Zongwill**
Probabilistic	**Monte Carlo**	**Pure Random, Chichinazde**
	Random Search	**Bremerman, adaptavive search from Matya, Beltrami and Indusi**
	Genetic Algorithm	**Selection, isolation, genes crossover, cromosoms mutation**
Hybrid	**Combination**	**Hartman, Torn, Faginoli, Gaviano**

- Classical interplanetary trajectory design is done by combining different type of arcs:
  - Launch phase with hyperbolic escape departure
  - Classical heliocentric arcs between planetary flybies
  - Powered or unpowered planetary swingbies or gravity assist trajectories
  - Singular transfer arcs (180º or 360º transfer angle)
  - Weak Stability Boundary Transfer techniques
  - Delta-V Leveraging techniques (Delta-VEGA)



- The Multiple Point Boundary Value Problem (MPBVP) is solved in 3 steps:
  - "**Patched Conics**" (trajectory with discontinuities)
  - "**Matched Asymptotic Expansions**" (continuous trajectory)
  - **Numerical integration** (parallel shooting + Newton method)



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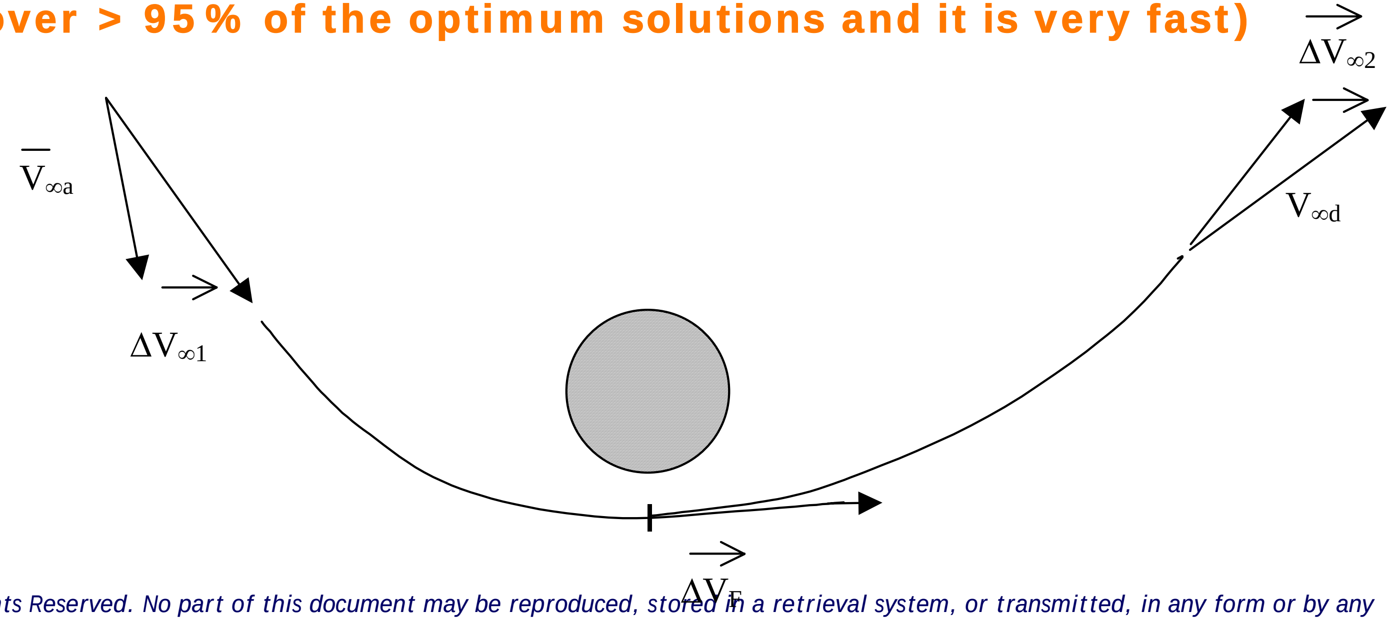
- **Lambert problem**: find the Keplerian trajectory joining two points in a given flight time.
- Lambert Theorem (1761): the transfer time depend on the sum of radii, c and the semimajor axis a.
- Numerical Formulations (iterative):
  - **Lambert-Euler method**
  - **Gauss method (1801)**
  - **Escobal (1965), C. Simó (1973), Battin (1987)**

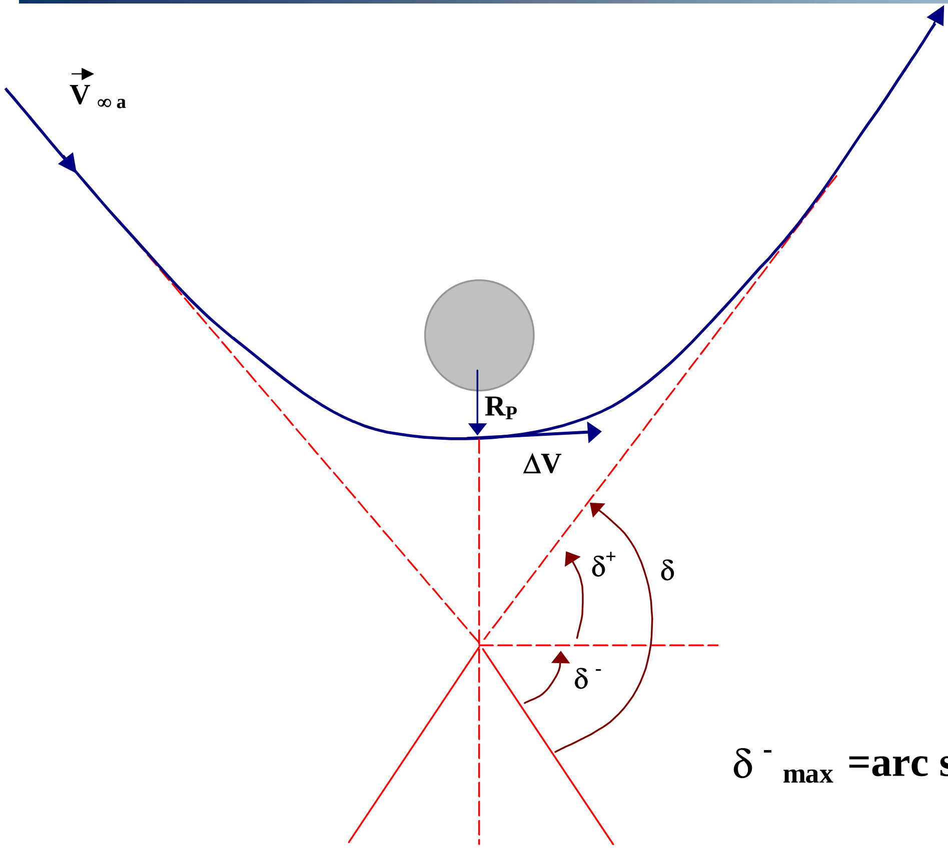
Powered Swing-by (I)

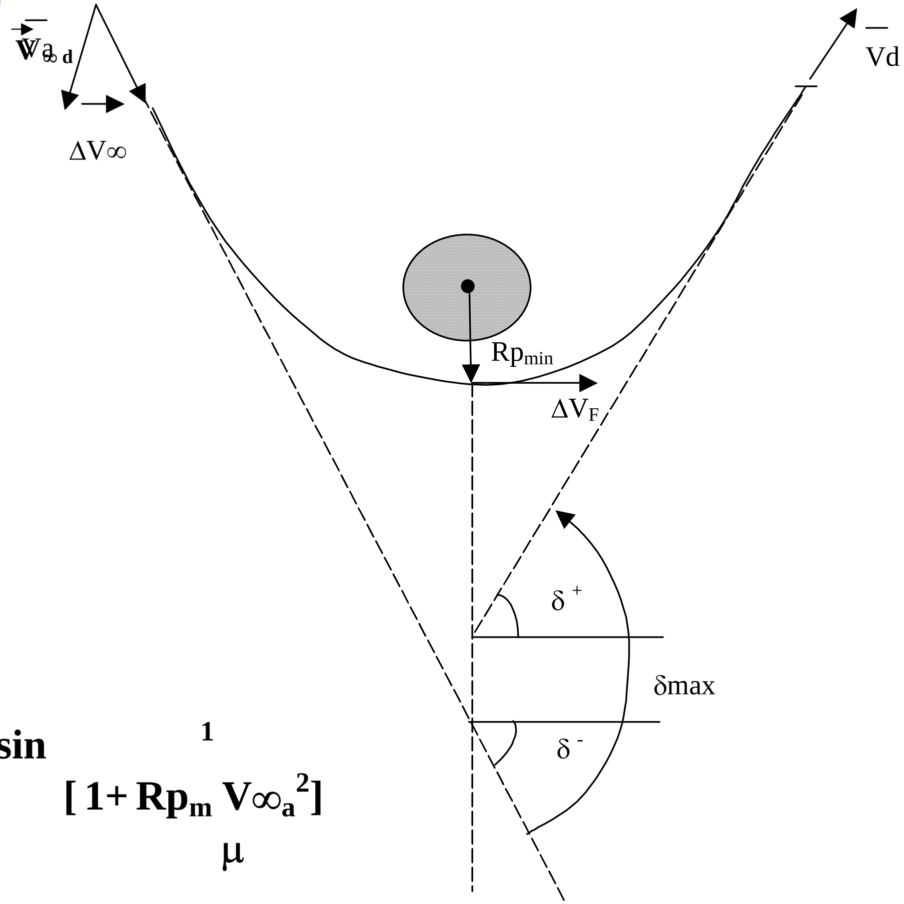
- If classical powered swing-by is considered:
  - **Manoeuvres at incoming or outgoing infinity plus**
  - **Manoeuvre at a finite distance (Type F)**
- Three options are available:
  - **Full optimisation for each case (CPU intensive)**
  - **3D interpolation in tabulated solutions**
  - **Simplified solution types F, ∞-F or F- ∞ with pericenter kick restriction (cover > 95% of the optimum solutions and it is very fast)**



Powered Swing-by (II)

- Computation of incoming Va and outgoing velocity Vd
- Computation of required deflexion angle δ
- Computation of maximum deflexion angle δ_{max}
  - **Max. deflexion from arrival leg $\delta_{max\ (-)}$**
  - **Max. deflexion from departure leg $\delta_{max(+)}$**
- If $\delta < \delta_{max}$: Pericenter kick solution, *root finder algorithm* in pericenter radius. Required ΔV and pericenter height are computed.
- If $\delta > \delta_{max}$: Pericenter kick solution at minimum pericenter altitude. Required ΔV and pericenter height are computed. Additional manoeuvre at:
  - **Incoming infinity (if Va < Vd)**
  - **Outgoing infinity (if Vd < Va)**

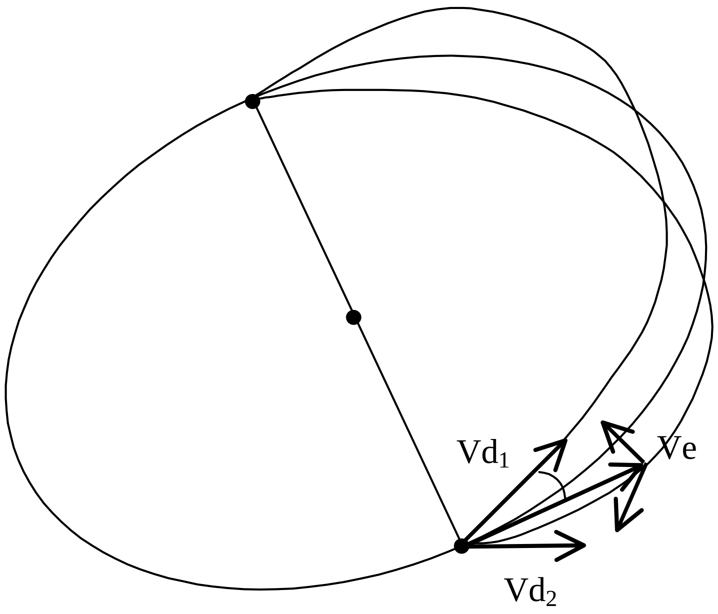


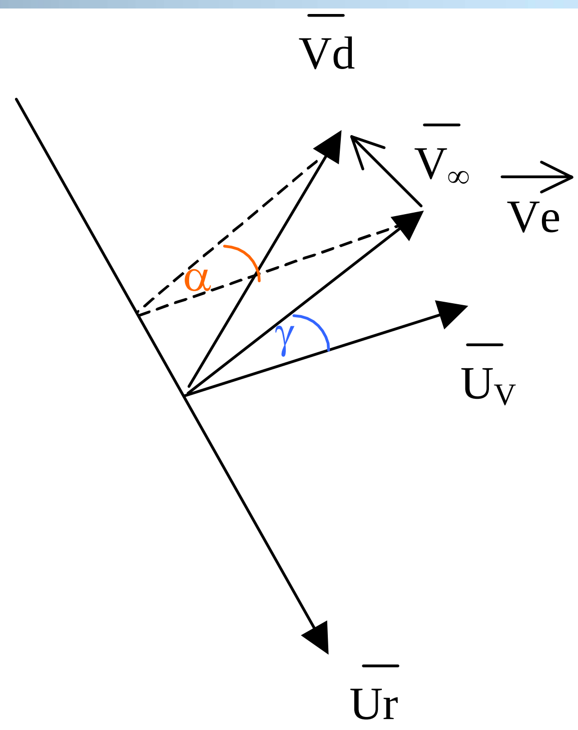


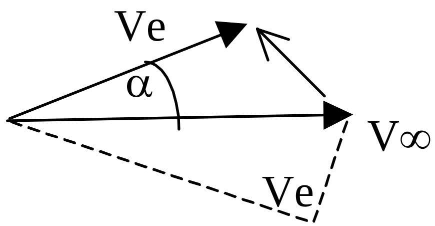
$$\delta^{-}_{max} = \arcsin \frac{1}{\left[1 + \frac{Rp_m V\infty_a^2}{\mu}\right]}$$

$$\delta^{+}_{max} = \arcsin \frac{1}{\left[1 + \frac{Rp_m V\infty_d^2}{\mu}\right]}$$

180º Singular Transfer



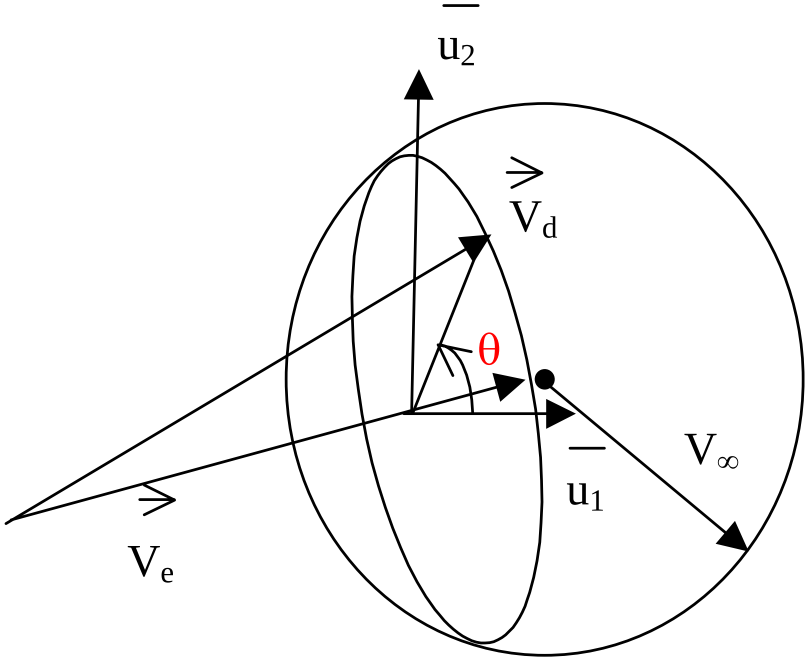




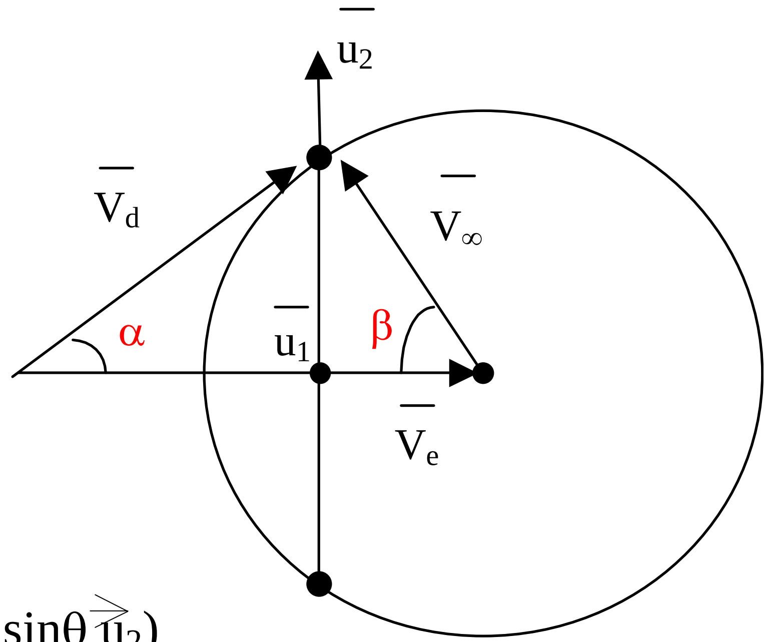
$$\alpha = 2 \arcsin \frac{V_\infty}{2*Ve*\cos\gamma}$$

$$\overline{Vd} = \overline{Ve} \pm V_\infty \cos \alpha/2 \, \vec{u}_n - V_\infty \sin \alpha/2 \, \vec{u}_v$$

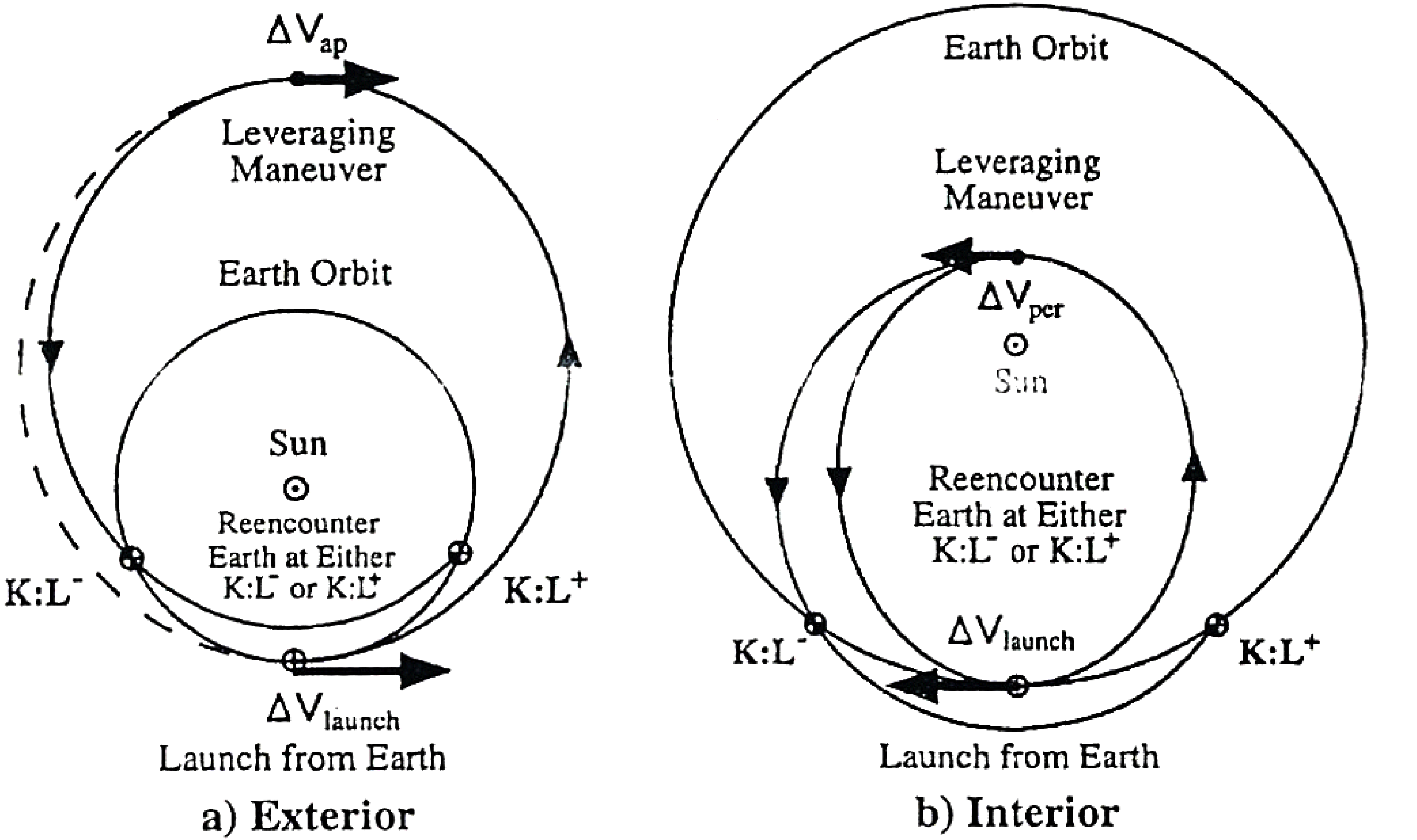
360º Singular Transfer



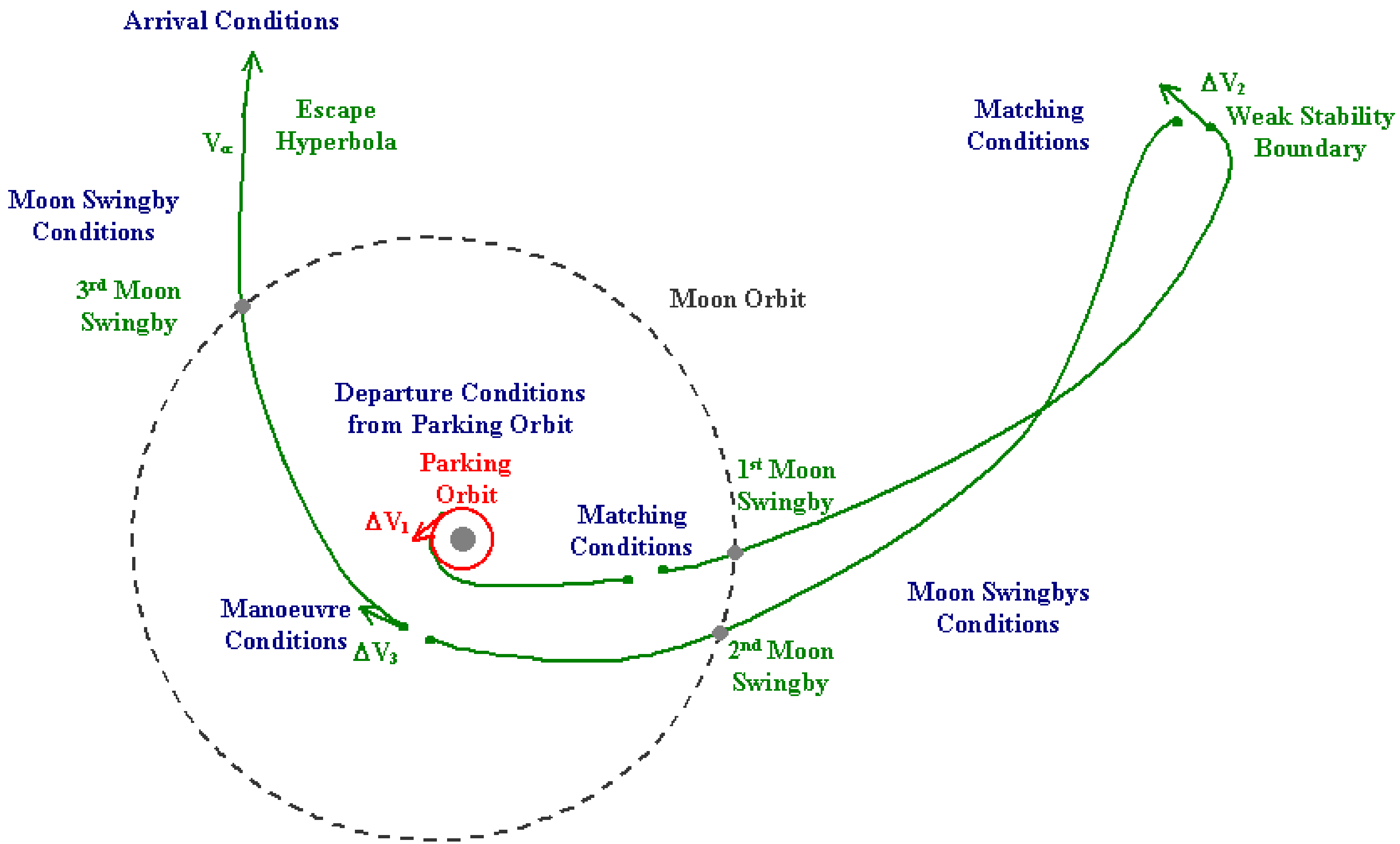
$$\cos\beta = \frac{V_e^2+V_\infty^2-V_d^2}{2\, V_e\, V_\infty}$$



$$\vec{Vd} = -V_\infty \cos\beta\, \vec{u}_{ve} + V_\infty \sin\beta\, (\cos\theta\, \vec{u}_1 + \sin\theta\, \vec{u}_2)$$


ΔV ap
Leveraging Maneuver
Earth Orbit
Sun
Reencounter Earth at Either K:L- or K:L+
K:L-
K:L+
ΔV launch
Launch from Earth
a) Exterior
Earth Orbit
Leveraging Maneuver
ΔV per
Sun
Reencounter Earth at Either K:L- or K:L+
K:L-
K:L+
ΔV launch
Launch from Earth
b) Interior

Interplanetary Trajectories: SLS, DLS, TLS



The optimal control problem is formulated as finding the optimum value of the control vector time function u and the parameters P in order to minimise the function:

$$J = \int_{t_0}^{t_f} L(x,u,P)dx + g(x(t_0), x(t_f), P)$$

subject to the boundary and path, equality and inequality constraints:

$$\dot{x} = f(x(t), u(t), P)$$

$$D_e(x(t_0), t_0, x(t_f), t_f, P) = 0, e = 1,..., k$$

$$D_i(x(t_0), t_0, x(t_f), t_f, P) \geq 0, i = k+1,..., r$$

$$C_e(x(t), u(t), P) = 0, e = 1,..., kk$$

$$C_i(x(t), u(t), P) \geq 0, i = kk+1,..., q$$

Variational Calculus or Pontryagin Principle is applied. The Hamiltonian and the adjoint vector are introduced such that:

$$\bar{J} = j + \upsilon^T D + \int_{t_0}^{t_f} (H - p^T \dot{x})dt$$

$$H = p^t f + \lambda^T C + L + \mu C$$

$$\dot{p}_j = -\frac{\partial H}{\partial x_j}, \; j = 1,\dots,n$$

$$\frac{\partial H}{\partial u_j} = 0, \; j = 1,\dots,m$$

$$\frac{\partial g}{\partial P_j} + \upsilon^T \frac{\partial D}{\partial P_j} = 0, \; j = 1,\dots,l$$

$$\left[\frac{\partial g}{\partial t} + \upsilon^T \frac{\partial D}{\partial j} \right](t_f) = 0$$

The transversatity conditions are:

$$p(t_0) = -\left[\frac{\partial g}{\partial x} + \upsilon^T \frac{\partial D}{\partial x}\right](t_0)$$

$$p(t_f) = -\left[\frac{\partial g}{\partial x} + \upsilon^T \frac{\partial D}{\partial x}\right](t_f)$$

For the inequality constraints:

$$\mu_i \geq 0, C_i(x,u,t) = 0, i = kk+1,\ldots,q$$

$$\mu_i = 0, C_i(x,u,t) \leq 0, i = kk+1,\ldots,q$$

There are discontinuities in the adjoints at the "<u>switching points</u>" between constrained and unconstrained regions.

- The resulting general constrained optimal control is a very complex Multiple Point Boundary Value problem:
  - State vector and adjoints differential equations shall be solved
  - Some variables are known at the initial, some at the final and some at the intermediate points of the trajectory
  - Optimality and transversality conditions provide the required relations to solve the mahtematical problem

OPTIMAL CONTROL PROBLEM

Indirect: exact mathematical solution of the MPBVP	**Gradient Restauration**	**Miele (Rice Un.,Houston), re-entry, aeroassisted**
	"Multiple Shooting"	**BNDCSO (Oberle, Grimm), low thrust trajectories**
Direct: constrained parameter optimisation NLP (discretisation)	**Direct Collocation**	**TROPIC (Hargraves, Jänsch): Hermite interpolation, ASTOS**
	"Multiple shooting" direct	**PROMIS (Bock, Plitt, Schnepper) direct multiple shooting scheme (ASTOS)**
	Finite Elements	**F. Bernelli-Zazzera, M. Vasile Polotecnico Milano**
Hybrids	**Biggs, Dixon (Hatfield)**	**Ariane 5 ascent , asteroid missions, comets, Mercury**

OPTIMAL CONTROL PROBLEM

Step	Objective	Comments
1. Problem Formulation	•Identification of state x, control variables u and parameters P •Identification of equality and inequality constraints	•Scale factors for: –variables, –constraints and –cost function •Technical constraints
2.- Initial Guess of Solution	•Find a first trajectory: –Simplified control law –Use of direct method to start an indirect method	•Gradient methods are very sensitive to the initial guess
3.- Optimal Control Method	Identify the most suitable OC method: indirect, direct or hybrid.	Direct methods are simpler but only approximations of the optimum

For the solution of a particular problem (e.g. launcher ascent trajectory or interplanetary low thrust trajectory) the first step for the selection of the optimisation algorithm is the trade off between direct and indirect methods:

- Direct methods like the **collocation algorithm** (program TROPIC) or the **direct multiple shooting** (program PROMIS) present the following advantages:
  - ♦ there is no adjoint differential equations
  - ♦ very easy to implement
  - ♦ short "setup time" for model modifications
  - ♦ very robust with respect to the starting estimates

  On the other hand, directs method have the following disadvantages:
  - ♦ the necessary optimality conditions are not satisfied, therefore only approximated solutions are found
  - ♦ local optimality of the solution found cannot be ascertained
  - ♦ very large nonlinear programming problems are generated

- **Indirect methods** like the Gradient Restoration algorithm or the Multiple Shooting (program BOUNDSCO) have the following advantages:
  - ♦ they solve the posed optimal control problem exactly, as far as possible numerically
  - ♦ they reveal the structure of the optimal solution (bang-bang control for instance)

  However indirect methods present the following disadvantages:
  - ♦ they involve the solution of the adjoint differential equations
  - ♦ second order methods, such as the Multiple Shooting, requires a good starting estimation for the algorithm to converge
  - ♦ some algorithms require an analytical pre-analysis to compute the switching structure and a good initial guess of the adjoint variables

In case that the selection problem has been reduced to indirect methods, the second step would be a trade off between the most prominent representatives of first (**Gradient Restoration**) and second (**Multiple Shooting**) order algorithms:

- The Gradient Restoration algorithm has the following advantages:
  - ♦ it is very robust with respect to the initial estimates
  - ♦ a short "setup time" is required for model modifications
  - ♦ the information on the switching structure may be numerically generated

  The disadvantage of the Gradient Restoration algorithm is the **slow convergence** rate close to the final optimum solution, typical of a first order gradient method.
- The Multiple Shooting algorithm presents the following advantages:
  - ♦ it is a second order algorithm with good final convergence rate
  - ♦ control discontinuities are easily treated

  On the other hand, the Multiple Shooting algorithm has the following disadvantages:
  - ♦ a good initial guess of the solution must be provided
  - ♦ the switching structure of the solution must be known in advance

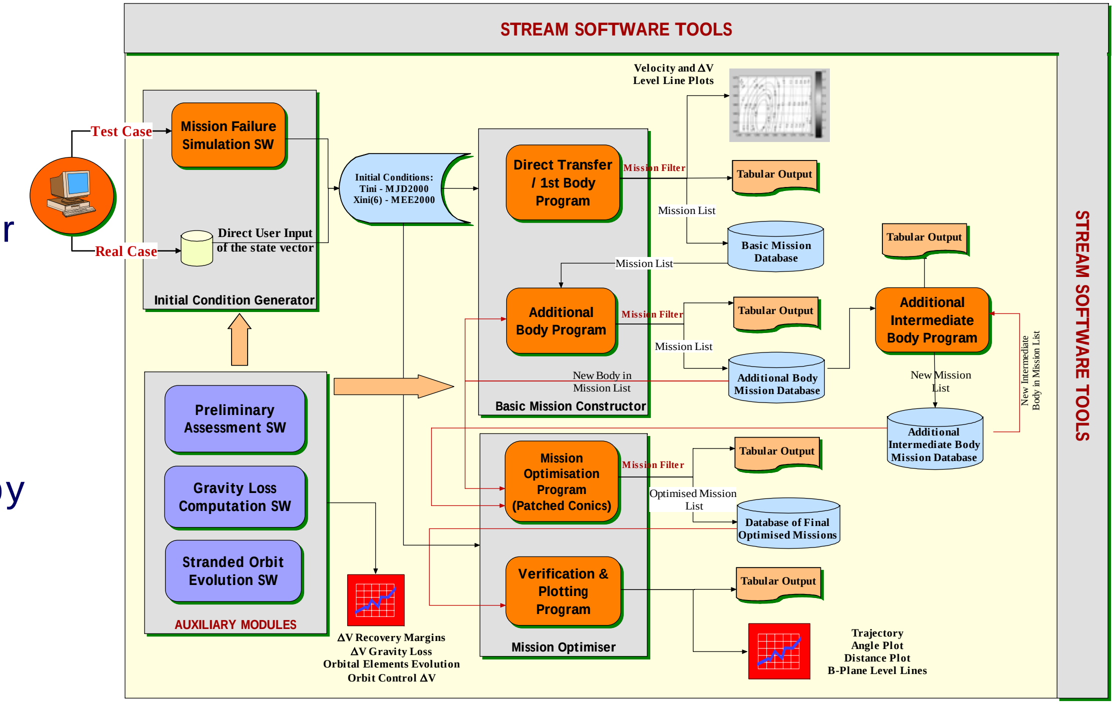
The Gradient-Restoration Algorithm has in addition the following advantages:

- This algorithm presents the most general formulation:
  - The functional to be minimized contains:
    - An integral part over the path
    - A function of the initial state vector
    - A function of the final state vector
  - The initial and final state vector may have:
    - A given value
    - A free value
    - Satisfy a set of relations
  - All kind of constraints may be applied by using suitable transformations:
    - State inequality constraints.
    - State equality constraints.
    - Problems with bounded control.
    - Problems with bounded state.
- The Gradient-Restoration is an algorithm suitable to be implemented in a modular way. The optimal control modules are independent of the functional modules which define each particular problem. Many different problems may be treated with a minimum change in the functional modules.
- An important property of this algorithm is that it produces a sequence of feasible suboptimal solutions; the functions obtained at the end of each cycle satisfy the constraints to a predetermined accuracy.

DEIMOS OPTIMISATION TOOLKIT

- **Systematic Search of Initial Guess:**
  - ⇨ Multiple Flyby Trajectory Generation: **MULFLY**
  - ⇨ Interplanetary Mission Recovery: **STREAM**
  - ⇨ Weak Stability Boundary Trajectories: **WATSBI**
- **Initial Trajectory Optimisation as NLP:**
  - ⇨ Optimisation without deep space manoeuvres: **OPTMIS**
  - ⇨ Optimisation with deep space manoeuvres: **OPTMAN**
- **Low Thrust and Rocket Ascent Trajectory Optimal Control Problems:**
  - ⇨ Gradient Restoration Algorithm: **GRANADA**
  - ⇨ Low Thrust Navigation: **LOTNAV**
- **Verification program: VERIF**
- **Generation of a database of solutions which are started by the initial guess generator, refined by the NLP and OCP packages and verified by VERIF**
- **The verification program provides also a powerful graphical result exploitation tool with trajectory and auxiliary parameter plotting capabilites**
- **Full Guidance, Navigation and Control capability is also provided**

- **Software Tool for Recovery of Escape Missions (STREAM):**
  - Design of Recovery Strategies in case of launcher underperformance or propulsion system failure
  - Implementation of different strategies for stranded orbits:
    - **Heliocentric**:
      - Multiple gravity assists
      - Delta-VEGA options
      - Singular arc transfer
    - **Earth-Moon**:
      - WSB transfers
      - Single, Double or Triple Lunar Swingby

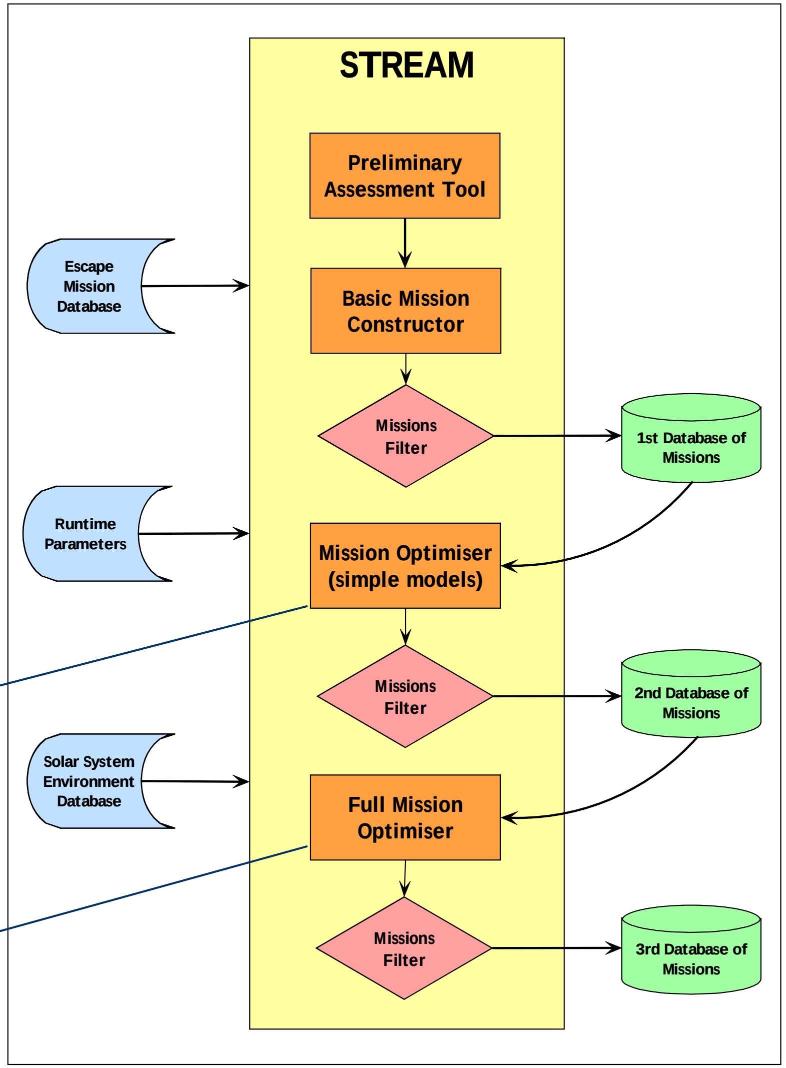


- **Direct Application to:**
  - Mars Express, Rosetta
  - Messenger

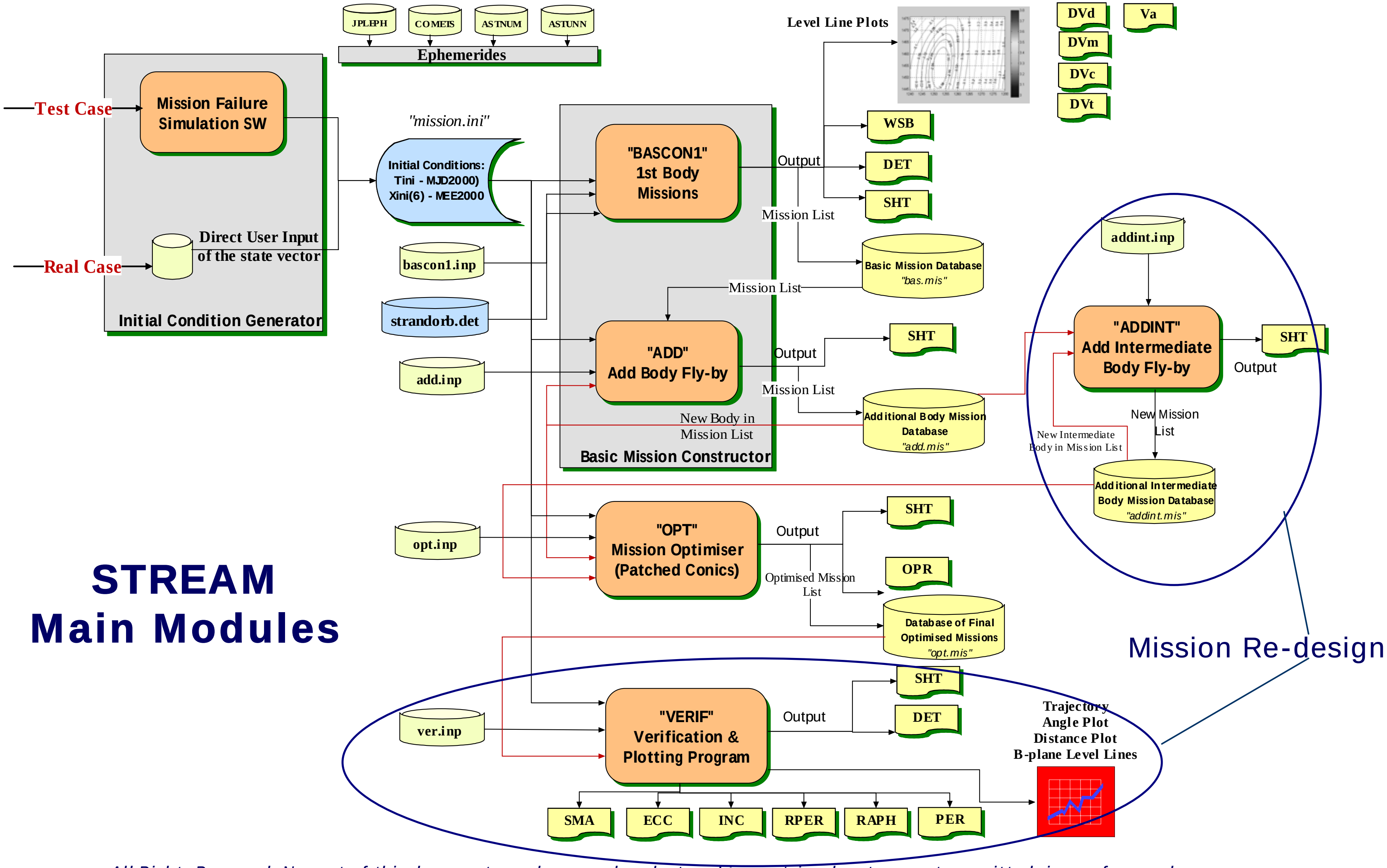
STREAM Recovery Software Tool: Functions and Interfaces

Direct Transfer Manoeuvres
Planetary Gravity Assists
Lunar Gravity Assists

Weak Stability
Boundary Transfers



Software Architecture and Data Flow (I)



STREAM Main Modules

- **Software Tool for Generation of Weak Stability Boundary Transfer Trajectories to Inner/Outer Planets and Natural Moons (WATSBI):**
  - Impulsive and Low Thrust propulsion are considered
  - Combination of:
    - **Multiple gravity assists**
    - **"Fuzzy Boundary" or WSB**

- **Direct application to:**
  - Venus Express
  - Mars Express
  - Bepi Colombo
  - Europa Orbiter
  - JIMO?

- **Software Tool for Low Thrust Guidance, Navigation and Control (LOTNAV):**
  - Covariance Analysis and Monte Carlo Simulation of interplanetary Low Thrust trajectories
  - Implementation of different control methods:
    - **Least-Squares approach**:
      - Deep Space 1 method
    - **Linear Quadratic Control**:
      - Bellman's control theory
    - **Trajectory reoptimisation**:
      - Indirect optimal control
- **Direct Application to:**
  - Bepi Colombo
  - Europa Orbiter
  - Solar Orbiter

ACT Problem Resolution (I)

- **A <u>preliminary analysis</u> of the ACT optimisation problem help to reduce the search field of the potential optimum solutions:**
  - **The cost function depends mainly on the product of the asteroid absolute velocity by the relative velocity, a maximum can be obtained in the following conditions:**
    - ⇨ Impact with the spacecraft in **RETROGRADE** orbit
    - ⇨ Impact at asteroid **PERICENTER** where its velocity is maximum
  - **Retrograde orbit can only be achieved by using a swingby in one of the outer planets:**
    - ⇨ Jupiter swingby is constrained by the minimum pericenter radius due to radiation
    - ⇨ Saturn swingby is more promising due to the higher deflection angle, a previous Jupiter flyby shall help to reach Saturn
  - **The optimum final sequence for the optimum trajectory should be then: <u>Jupiter – Saturn – Jupiter (to accelerate the probe) – asteroid at pericenter</u>**
  - **A reference value of the maximum objective function (<u>our</u> target) that can be reached is: 1500 kg (full mass with no fuel consumption) x 50 km/s (max. rel. vel. in a solar system non-escape trajectory) x 25 km/s (pericenter velocity) = $1875 \cdot 10^6$ kg $\cdot$ km^2 / s^2**
  - **Our target is to get as close as possible to $1875 \cdot 10^6$ kg $\cdot$ km^2 / s^2**

ACT Problem Resolution (II)

- **Due to the very limited manpower available (3 weekends) to solve this problem, An important simplification of Deimos Optimisation Toolkit was mandatory:**
  - **Systematic Search of Initial Guess:**
    - ⇨ Only Multiple Flyby Trajectory Generation with powered swingbies was considered: **MULFLY**
    - ⇨ The use of Delta-VEGA and Weak Stability Boundaries techniques with **STREAM** and **WATSBI** was discarded due to time and manpower limitations
  - **Initial Trajectory Optimisation as NLP:**
    - ⇨ Only optimisation without deep space manoeuvres: **OPTMIS**
    - ⇨ The incorporation of intermediate deep space manoeuvres with the package **OPTMAN** was discarded due to time constraints
  - **Full Low Thrust Optimal Control Problems:**
    - ⇨ Low Thrust Trajectory Generation: **LOTNAV**
    - ⇨ The full OCP optimisation with the Gradient Restoration algorithm (**GRANADA**) was not performed due to time and manpower limitation
  - **The verification program VERIF was used**

ACT Problem Resolution (III)

- This process has been based on the following sub-processes and procedures:
  - Use of multiple planetary swingbys
  - Inclusion of regular and singular (180° and 360° transfers at the same body) swingbys
  - Assumption of discrete manoeuvres at the arrival branch or at the departure branch of the swingby hyperbolas
  - Scanning of solutions by means of a Lambert Solver with several number of complete revolutions
  - Discrimination of the constraint compliant and best solutions, rule out of the rest
  - Non-linear programming optimisation of the best suited cases
- The intense and complete scanning of solutions ensures that the best optimal cases are not missed in the process, which solutions are improved by means of a local non-linear programming optimiser.
- Making use of such approach, the following types of trajectories were tested:
  - EVEMEJSA
  - EVEVEVEJSA
  - EVVEEVVEVEJSA
  - Etc.

Obtained Solutions (I)

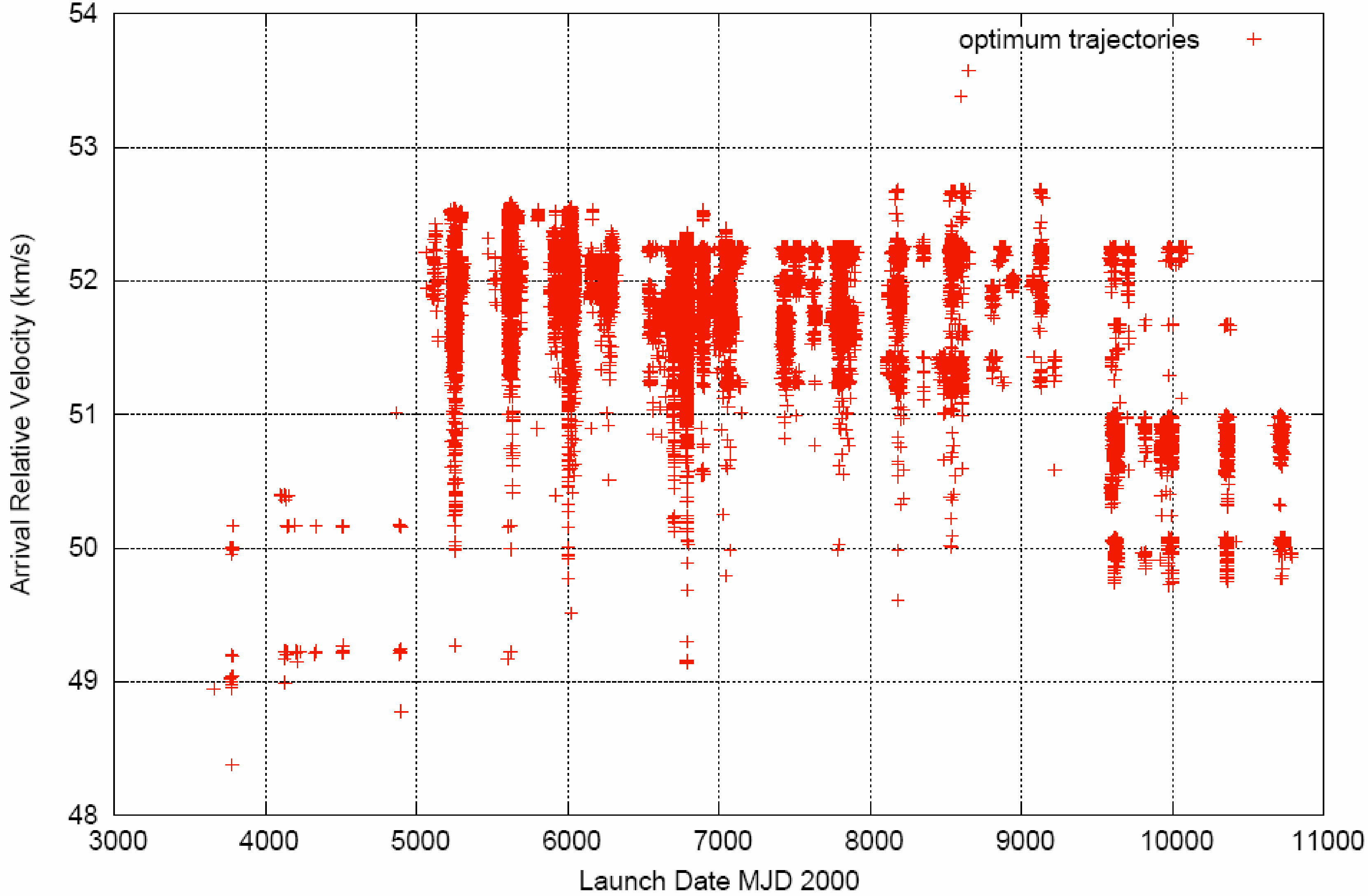
- **The proposed strategy allowed obtaining myriads of solutions within the proposed conditions, which at the same time formed families of solutions with similar performances.**

- **Strategies were based on using:**
  - **Departure from Earth and final arrival at 2001 TW22**
  - **Multiple swingbys at the proposed bodies (with the exception of Mercury)**
  - **Singular transfers at the same massive body in some instances**

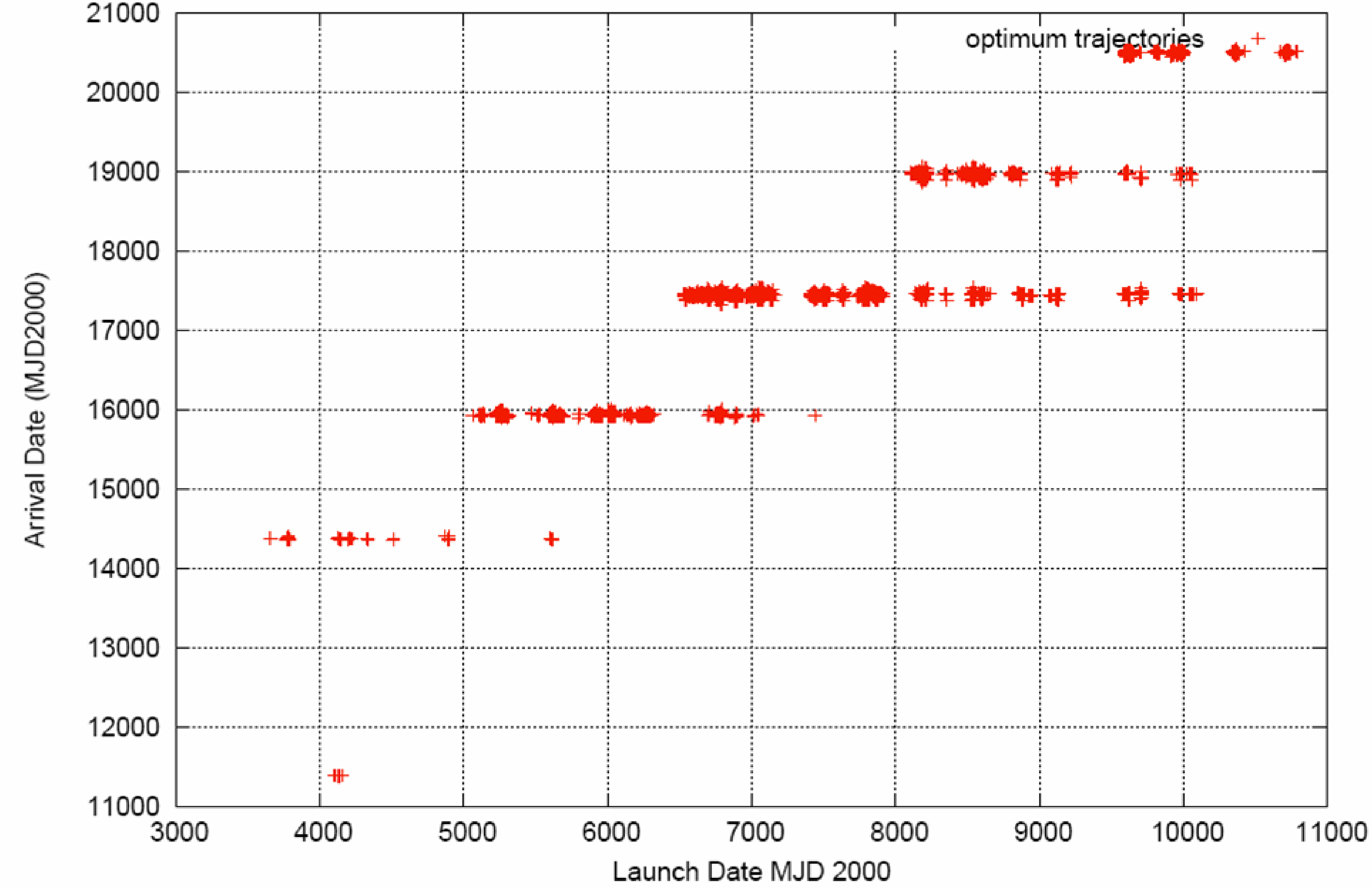
- **In summary, almost 47,000 solutions have been found by means of this scanning procedure with a cost function above $1.6 \cdot 10^6$ $kg \cdot km^2 / s^2$.**

Obtained Solutions (II)

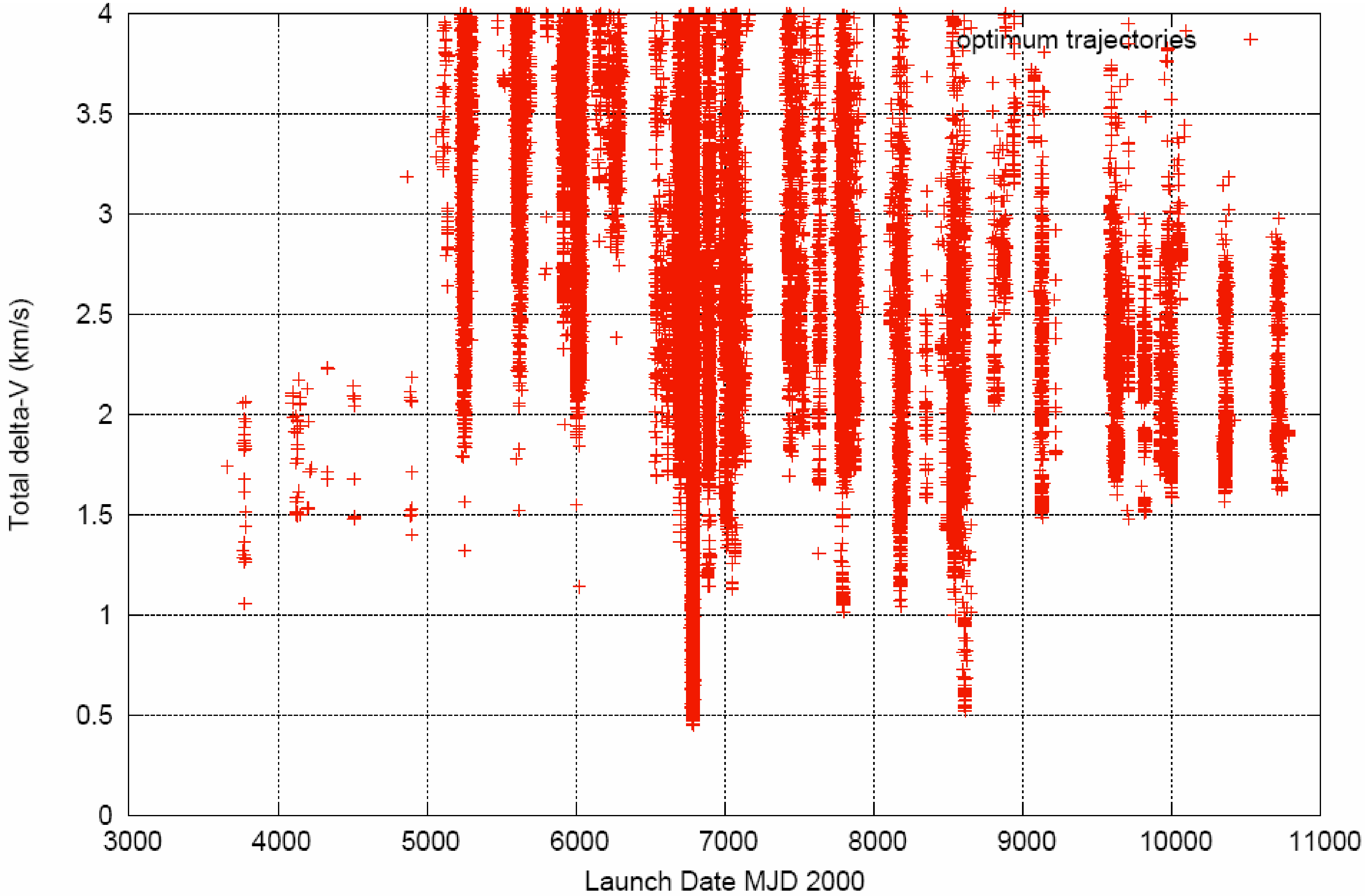
NMIS	TD	VDEP	NFB	TA	VA	DUR	DVT	OBJ
01	4125.83092	2.703	7	11392.23639	50.390	7266.4	1.502	1.654
02	4123.52933	2.707	7	11392.08947	50.390	7268.6	1.565	1.650
03	4125.42236	2.702	7	11392.38328	50.391	7267.0	1.502	1.654
04	4125.82525	2.703	7	11392.24719	50.390	7266.4	1.501	1.654
05	4128.41736	2.720	7	11392.28907	50.390	7263.9	1.514	1.653
06	4125.93121	2.713	7	11396.76536	50.396	7270.8	1.587	1.648
07	4133.32497	2.737	7	11388.64402	50.383	7255.3	1.830	1.632
08	4095.44543	3.200	7	11392.08539	50.390	7296.6	2.095	1.614
...								
46938	9960.18038	2.830	10	20494.09219	50.735	10533.9	2.192	1.623
46939	9955.86339	2.809	10	20495.35111	50.716	10539.5	2.065	1.632
46940	9956.79798	2.809	10	20493.03835	50.731	10536.2	2.054	1.634
46941	9955.79388	2.812	10	20498.39949	50.687	10542.6	2.053	1.632
46942	9960.42043	2.840	10	20488.19640	50.758	10527.8	2.381	1.611
46943	9955.21495	2.815	10	20496.84143	50.703	10541.6	2.060	1.632
46944	9955.93604	2.808	10	20489.04768	50.748	10533.1	2.064	1.633
46945	9956.97116	2.809	10	20490.52630	50.743	10533.6	2.058	1.634
46946	9955.69687	2.809	10	20491.80815	50.739	10536.1	2.090	1.632
46947	10718.31875	2.881	10	20491.92369	50.737	9773.6	2.492	1.605
46948	10713.82091	2.877	10	20490.54041	50.743	9776.7	2.468	1.607


optimum trajectories
54
53
52
51
50
49
48
3000
4000
5000
6000
7000
8000
9000
10000
11000
Arrival Relative Velocity (km/s)
Launch Date MJD 2000

Obtained Solutions



Obtained Solutions



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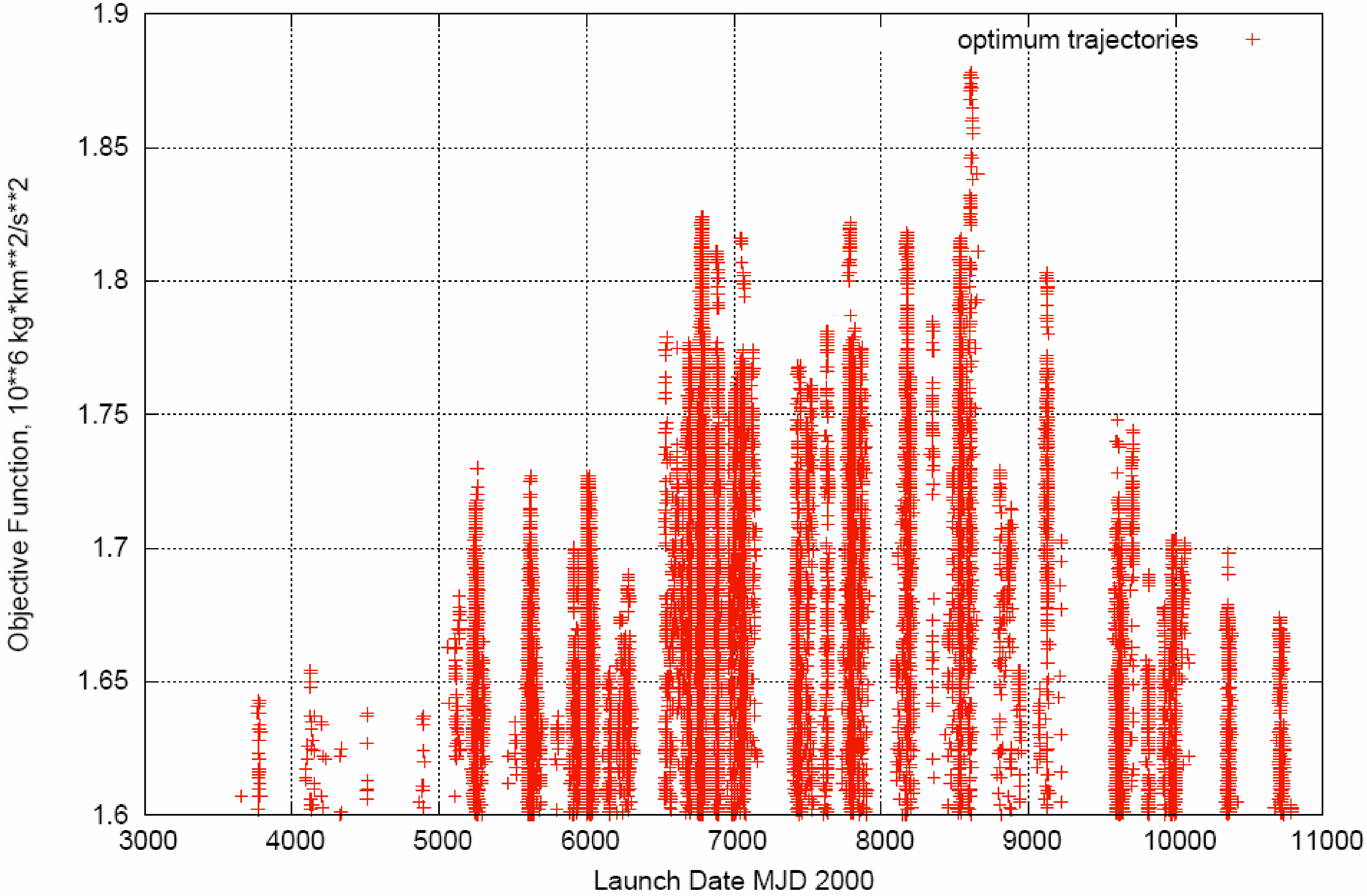
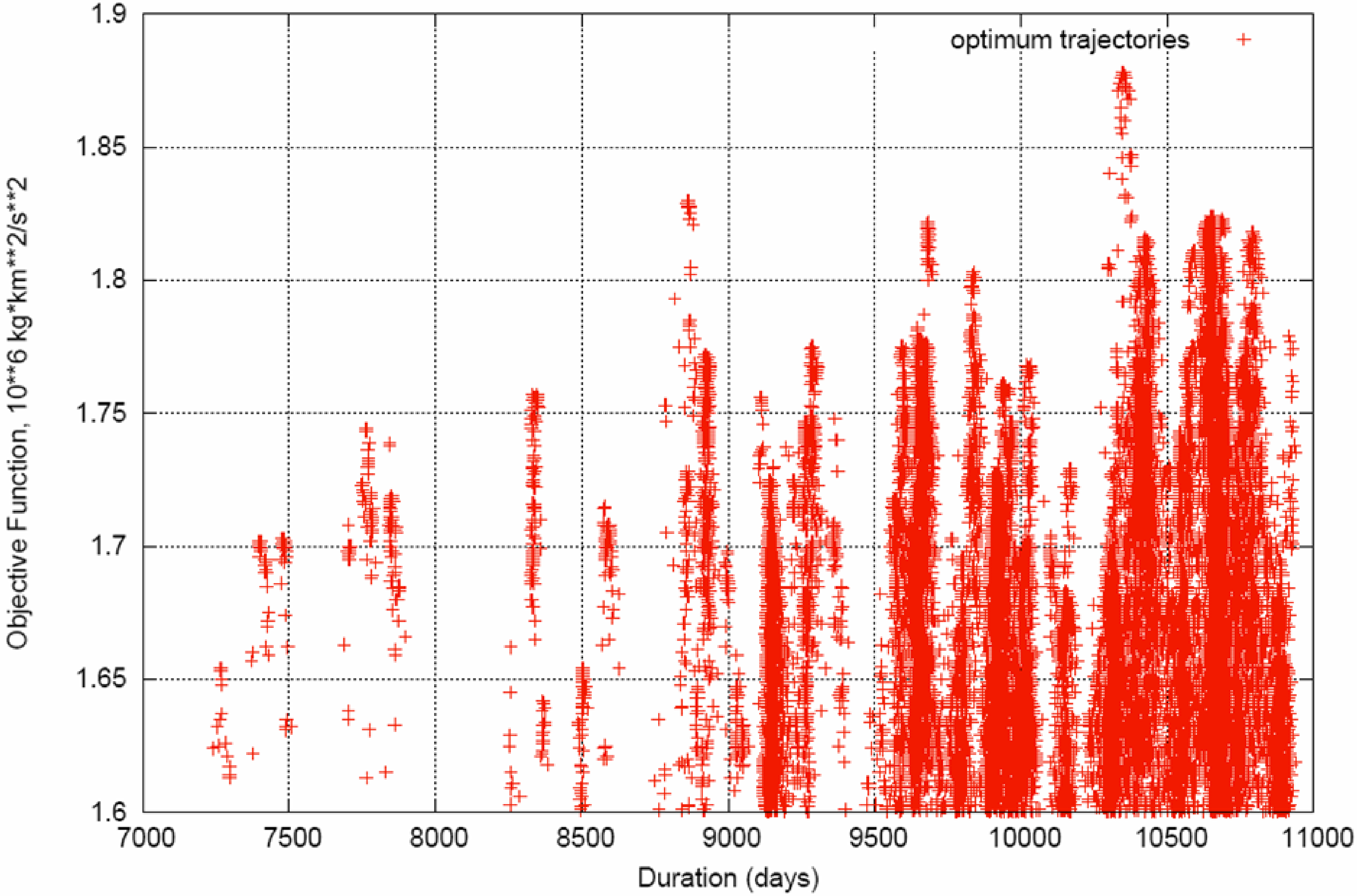
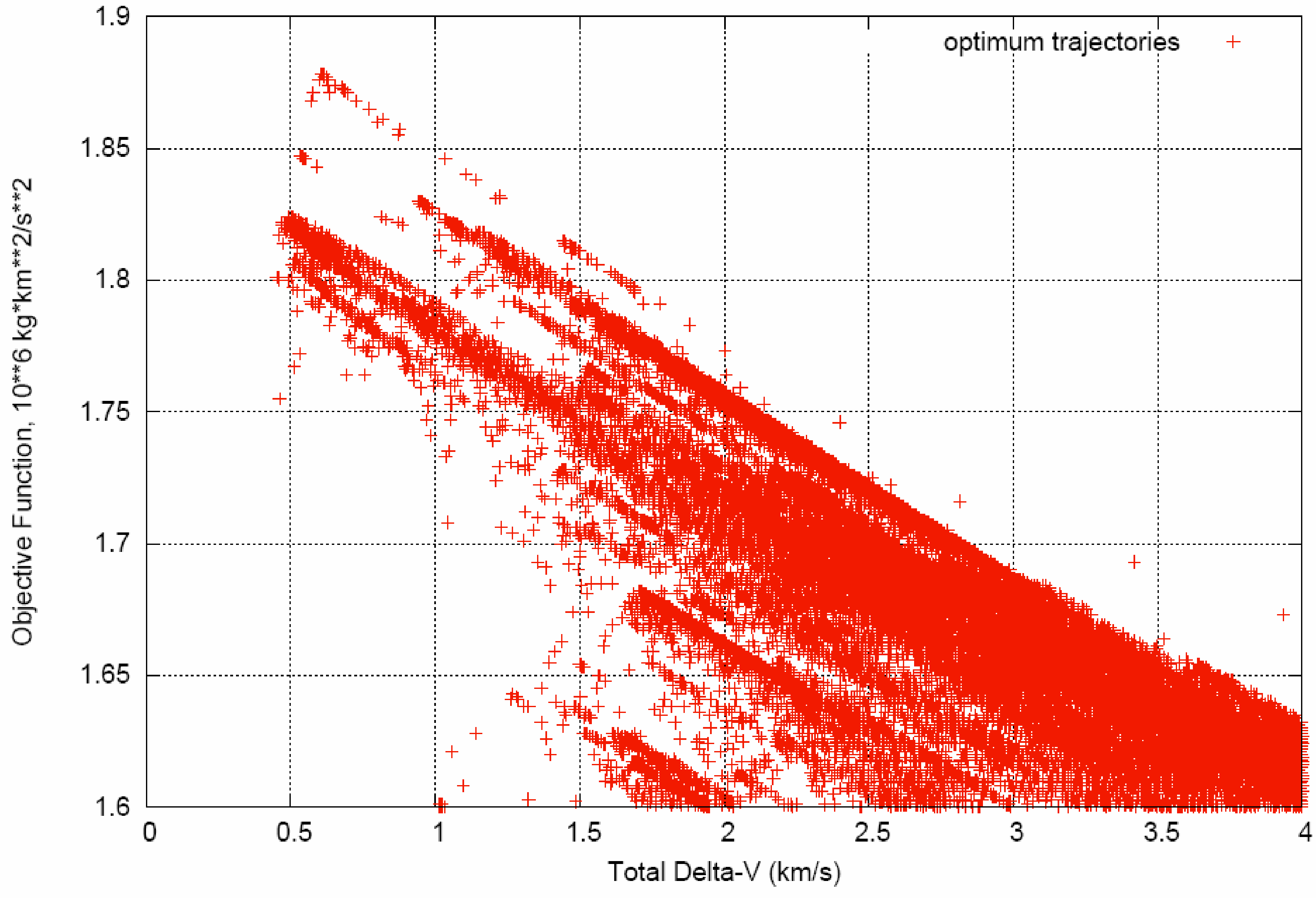


Obtained Solutions



Obtained Solutions



Selected Solutions

Mission	Profile	Departure date (MJD2000)	Arrival date (MJD2000)	Ballistic cost function (kg km^2/s^2)	Chemical delta-V (km/s)
A (case 3772)	EVEVEJSA	6748.5	17423.0	1.753	1.414
B (case 4698)	EVEVEJSA	6786.9	17466.5	1.739	2.227
C (case 12822)	EVEVEJSA	7786.2	17441.0	1.729	1.870
D (case 50B)	EVEMEJSA	9616.1	17461.3	1.739	2.252
E (case 4662C)	EVEVEVEJSA	6786.1	17421.2	1.822	0.482
F (case 43229D)	EVVEEVVEVEJSJA	8608.6	18956.1	1.878	0.613

Obtained Solutions: CASE E

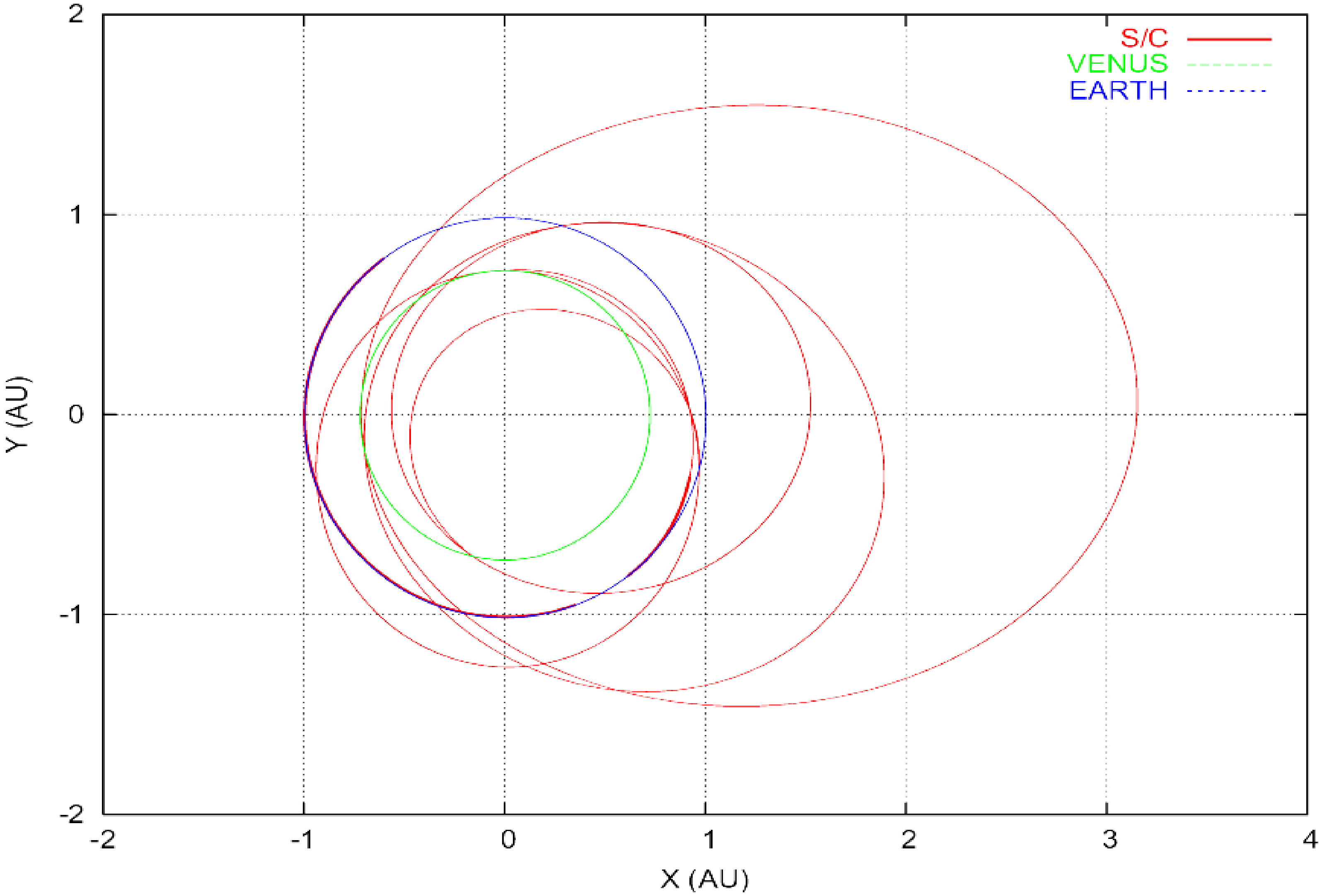
- **Case E had the following manoeuvres obtained in the Systematic Ballistic Search:**
  - **215 m/s at Earth departure**
  - **267 m/s prior to arriving to Jupiter**

- **The first delta-V is too high to be implemented by the low-thrust system in the short arc between Earth and Venus, thus it was decided to launch six months before and perform an Earth swingby. Such strategy allowed obtaining convergence in the proposed area with the low-thrust manoeuvre split just after launch and after the Earth swingby.**

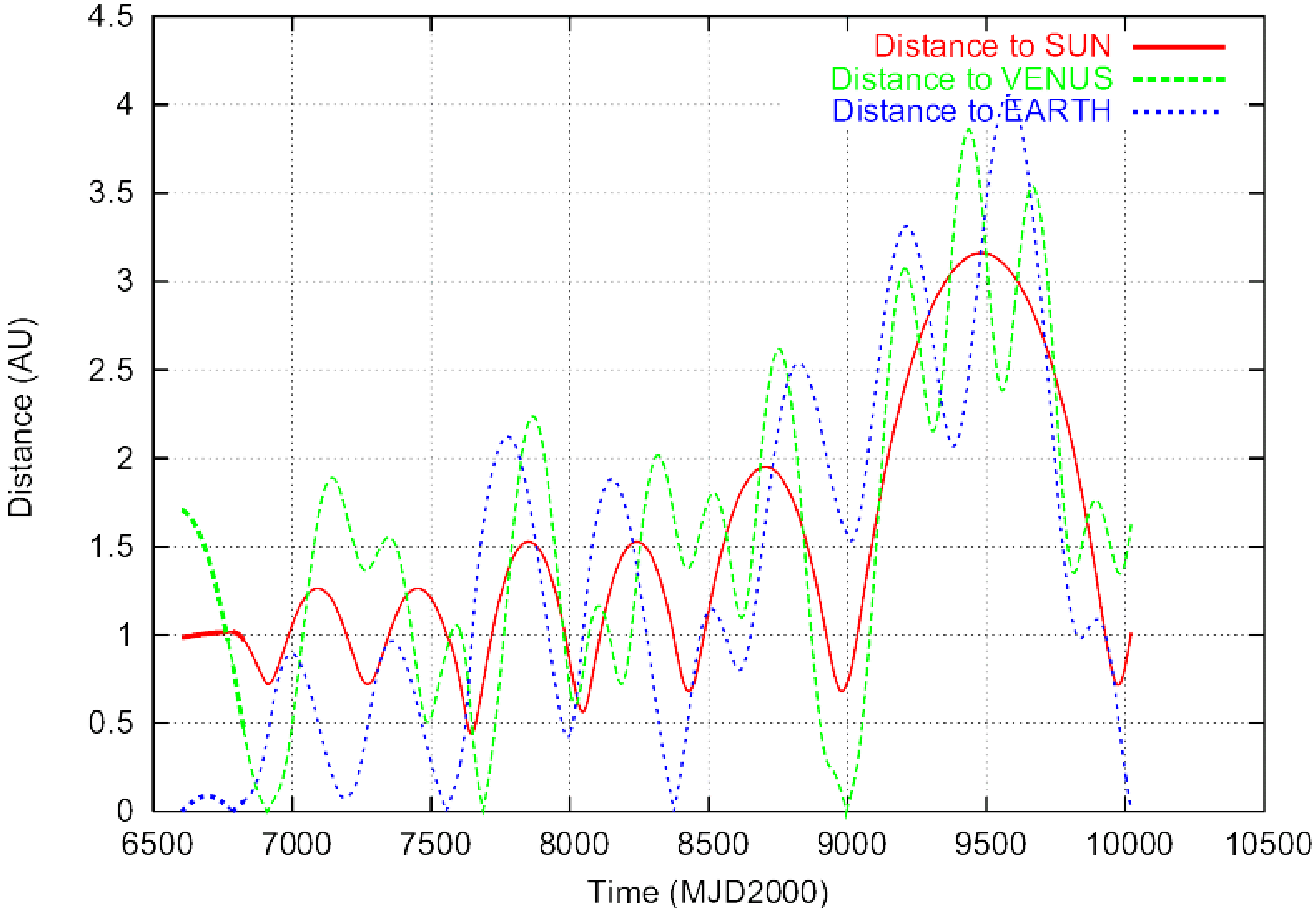
- **With the mentioned addition, the solution structure is finally:**
  - **E0-T-C-E1-T-C-V1-C-E2-C-V2-C-E3-C-V3-C-E4-C-T-J-C-S-C-A**

- **The final obtained cost function for this case is:**
  - **Cost Function = $1.789510 \cdot 10^6$ kg · km2 / s2**

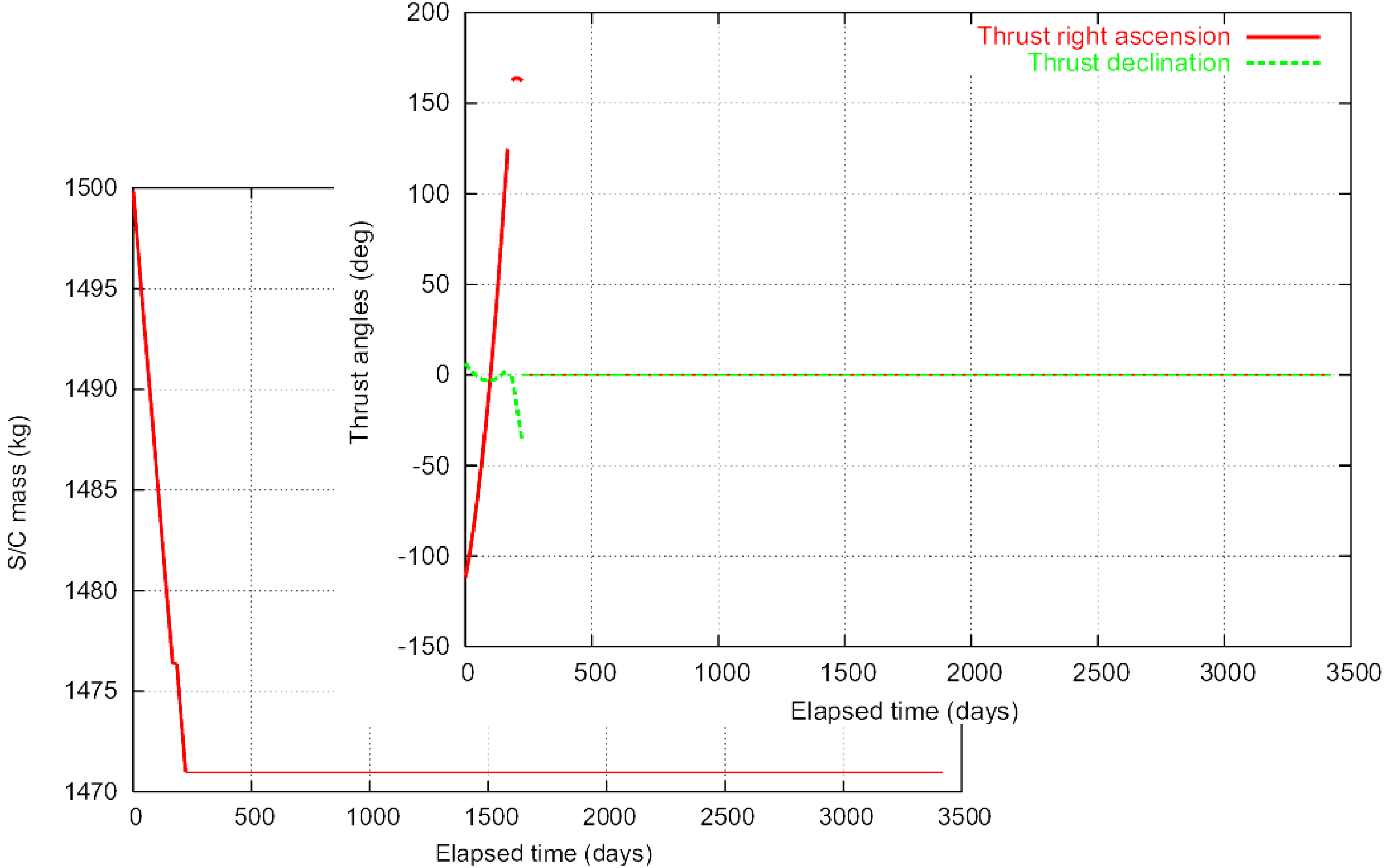
Obtained Solutions: CASE E



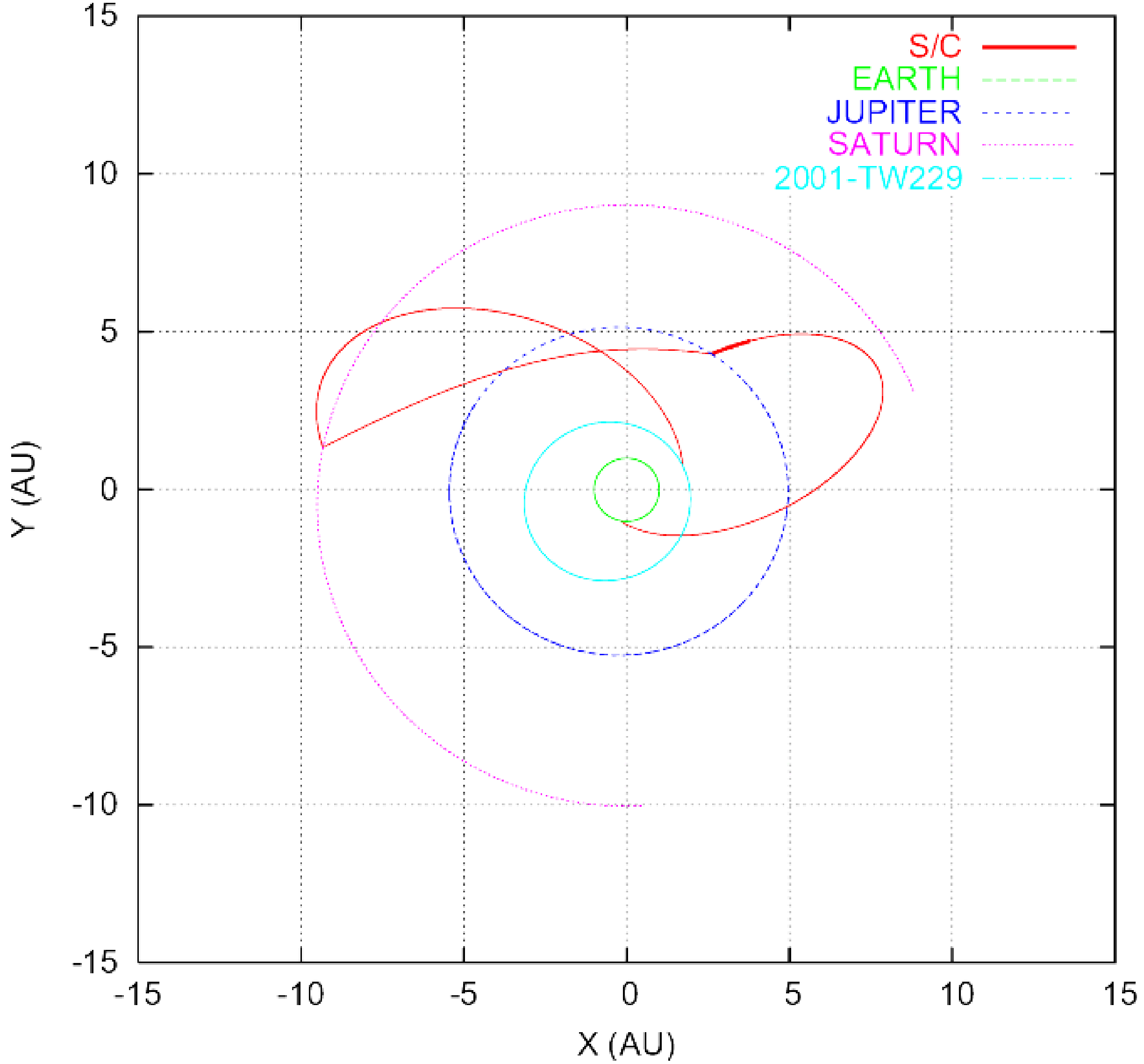
Obtained Solutions: CASE E



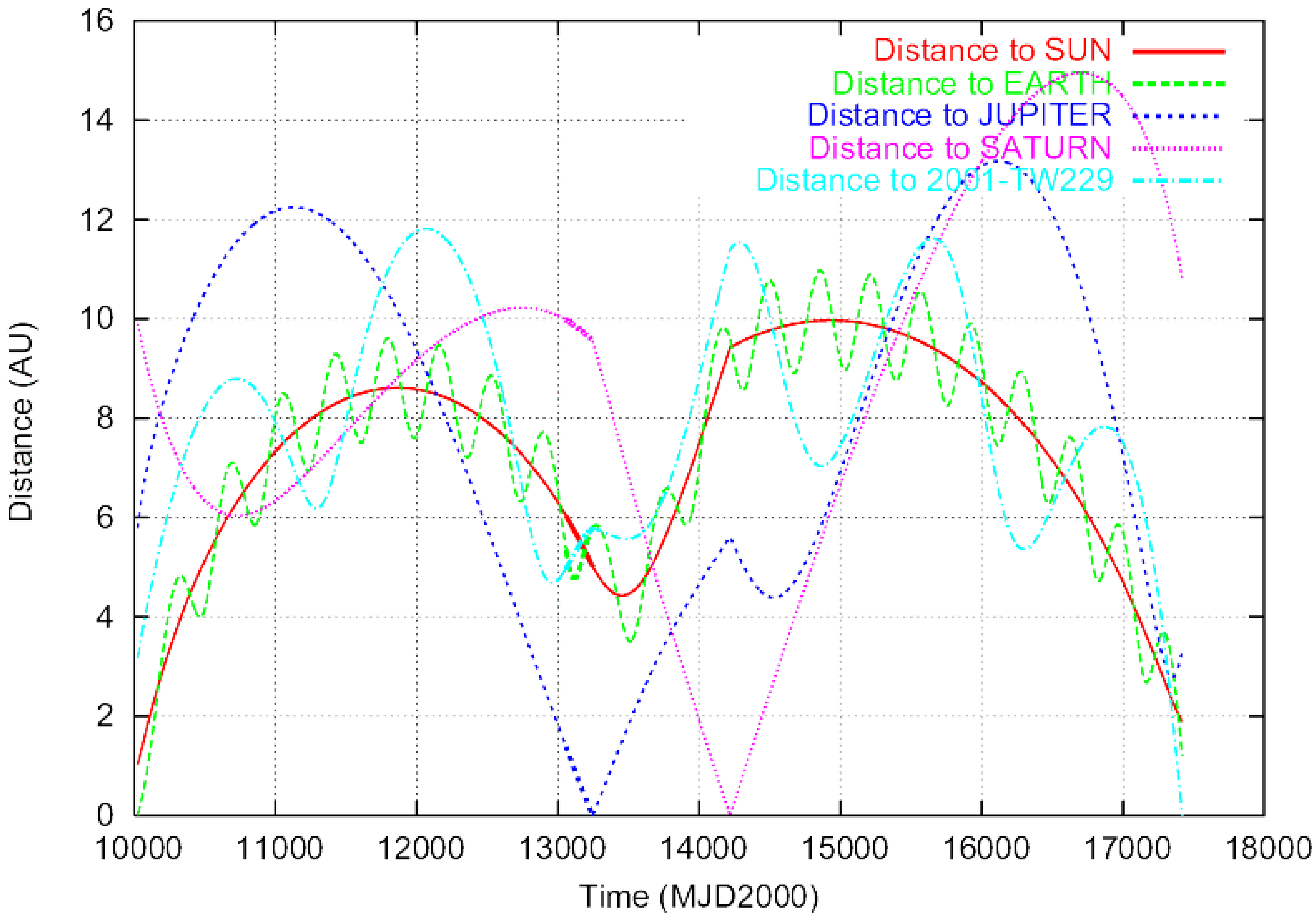
Obtained Solutions: CASE E



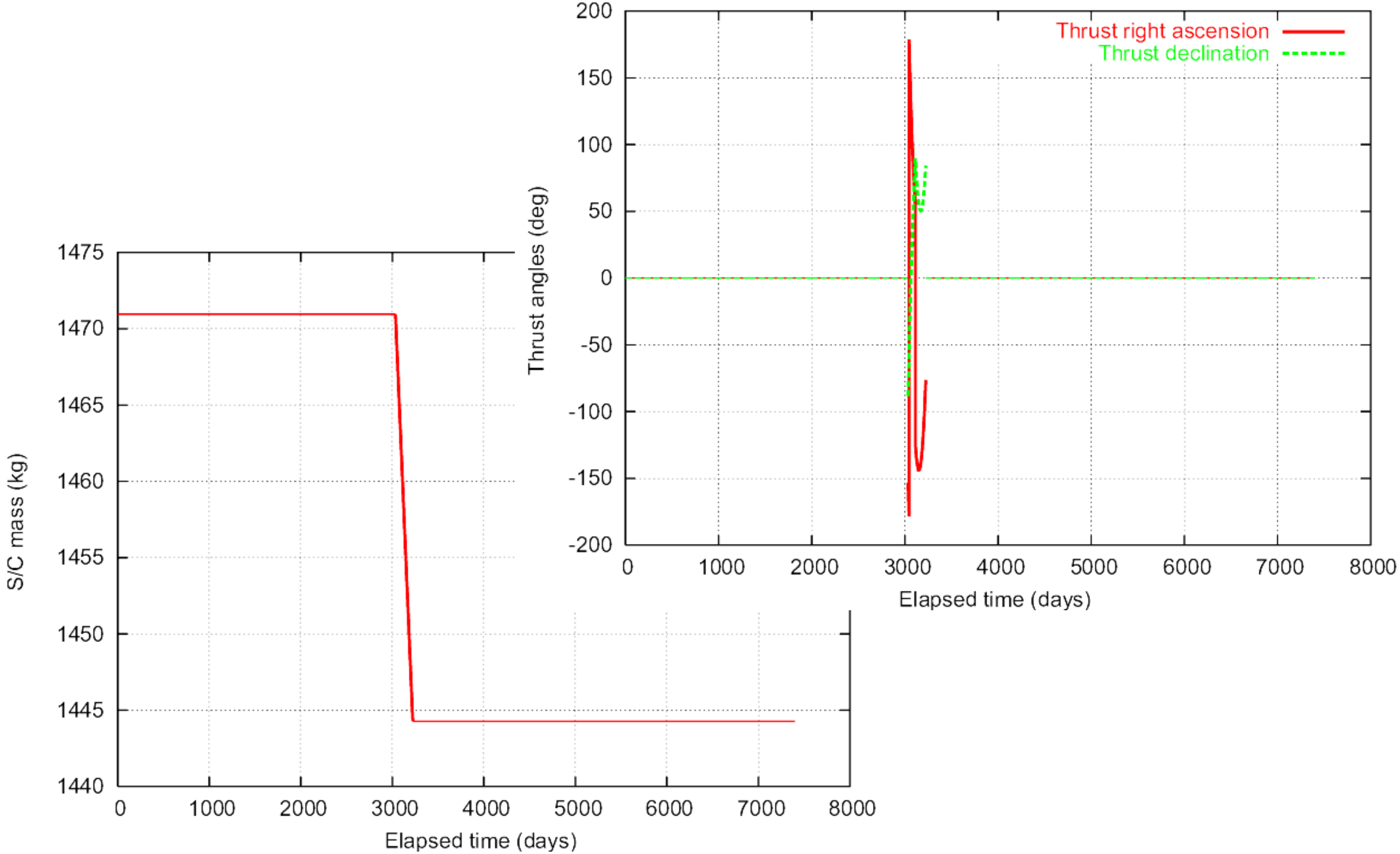
Obtained Solutions: CASE E



Obtained Solutions: CASE E



Obtained Solutions: CASE E



Obtained Solutions: CASE E

MAIN EVENT	EVENT DATE	THRUST SWITCH	ARCS	DURATION (days)
Departure	26/01/2018 14:38:13	ON	Thrust	167.4784
Thrust switch	13/07/2018 02:07:03	OFF	Coast	17.3794
Earth swingby	30/07/2018 11:13:21	ON	Thrust	38.576
Thrust switch	07/09/2018 01:02:43	OFF	Coast	85.506
Venus swingby	01/12/2018 13:11:21	---	Coast	645.0139
Earth swingby	06/09/2020 13:31:25	---	Coast	132.4091
Venus swingby	16/01/2021 23:20:30	---	Coast	687.5617
Earth swingby	05/12/2022 12:49:20	---	Coast	621.4137
Venus swingby	17/08/2024 22:45:02	---	Coast	1025.9896
Earth swingby	09/06/2027 22:30:03	---	Coast	3034.6637
Thrust switch	30/09/2035 14:25:44	ON	Thrust	189.4053
Jupiter swingby	07/04/2036 00:09:24	OFF	Coast	972.631
Saturn swingby	05/12/2038 15:18:07	---	Coast	3202.5692
Arrival to asteroid	12/09/2047 04:57:47	---	End	

Obtained Solutions: CASE E

	Epoch (MJD2000)	S/C Mass (Kg)	Arrival velocity X-Component (km/s)	Arrival velocity Y-Component (km/s)	Arrival velocity Z-Component (km/s)	Arrival velocity Modulus (km/s)	Flyby Radius (km)
Earth departure	6600.610	1500.00	0.03964	1.20307	-2.19113	2.50000	
Earth swingby 1	6785.468	1476.40	0.06995	-1.16643	2.35545	2.62937	36201.4
Venus swingby 1	6909.550	1470.96	-2.47155	-1.94990	-3.88806	5.00277	21889.6
Earth swingby 2	7554.564	1470.96	-7.68564	0.64809	0.74554	7.74887	10488.7
Venus swingby 2	7686.973	1470.96	-5.66290	-13.02923	-3.40751	14.60960	7371.6
Earth swingby 3	8374.534	1470.96	-1.37294	-13.05499	-5.33917	14.17125	15102.9
Venus swingby 3	8995.948	1470.96	-6.01486	-7.47798	-3.17862	10.10952	8920.3
Earth swingby 4	10021.938	1470.96	-1.10603	-14.72756	-8.39462	16.98805	8102.9
Jupiter swingby 1	13246.007	1444.27	-0.42394	-11.43879	-4.69741	12.37300	634039.3
Saturn swingby 1	14218.638	1444.27	-15.23663	0.32662	0.51792	15.24892	71840.0
2001-TW229 arrival	17421.207	1444.27	12.95700	-47.45967	-15.74819	51.65568	

Obtained Solutions: CASE F

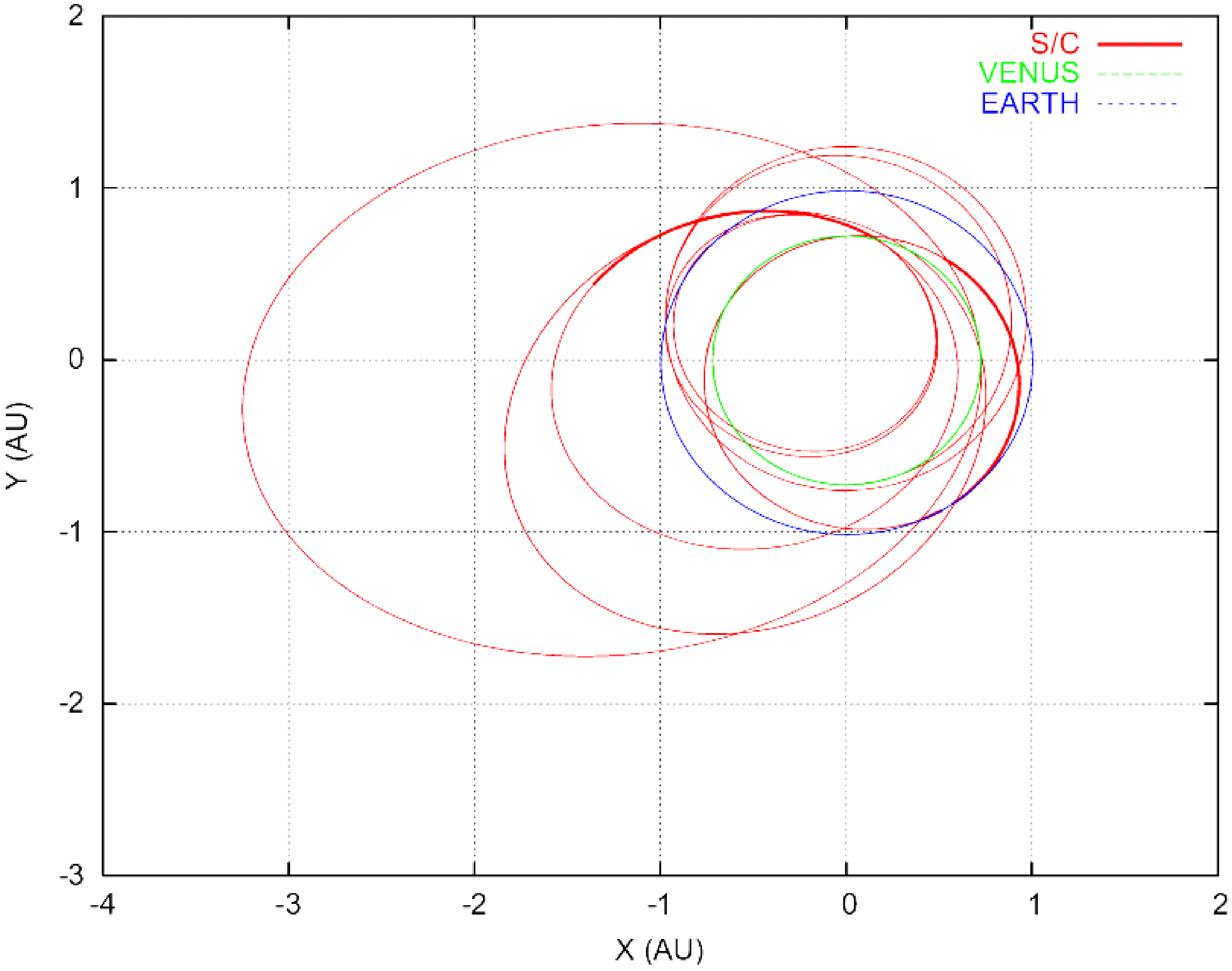
- **Case F had the following manoeuvres obtained in the Systematic Ballistic Search:**
  - **181 m/s at Earth departure**
  - **104 m/s after V4**
  - **40 m/s after E3**
  - **287 m/s after E4**

- **As it can be observed it is a complex trajectory profile with:**
  - **4 Earth swingbys**
  - **5 Venus swingbys**
  - **2 Jupiter swingbys**
  - **1 Saturn swingby**

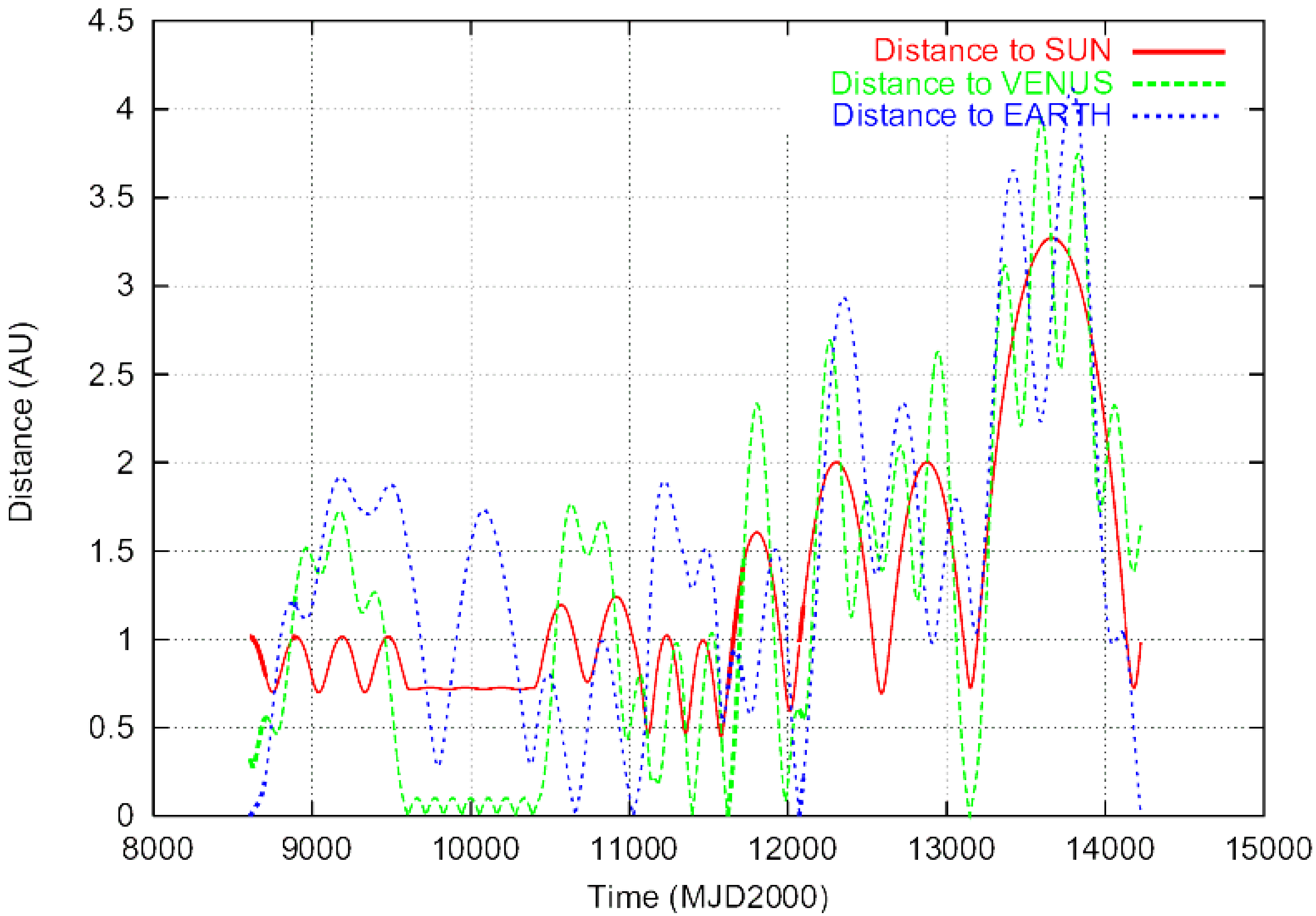
- **The solution structure is:**
  - **E0-T-C-T-C-V1-C-V2-C-E1-C-E2-C-V3-C-V4-T-C-E3-T-C-V5-C-E4-T-C-T-J1-C-S-C-J2-C-A**

- **The final obtained cost function for this case is:**
  - **Cost Function = $1.819872 \cdot 10^{6}$ kg · km2 / s2**

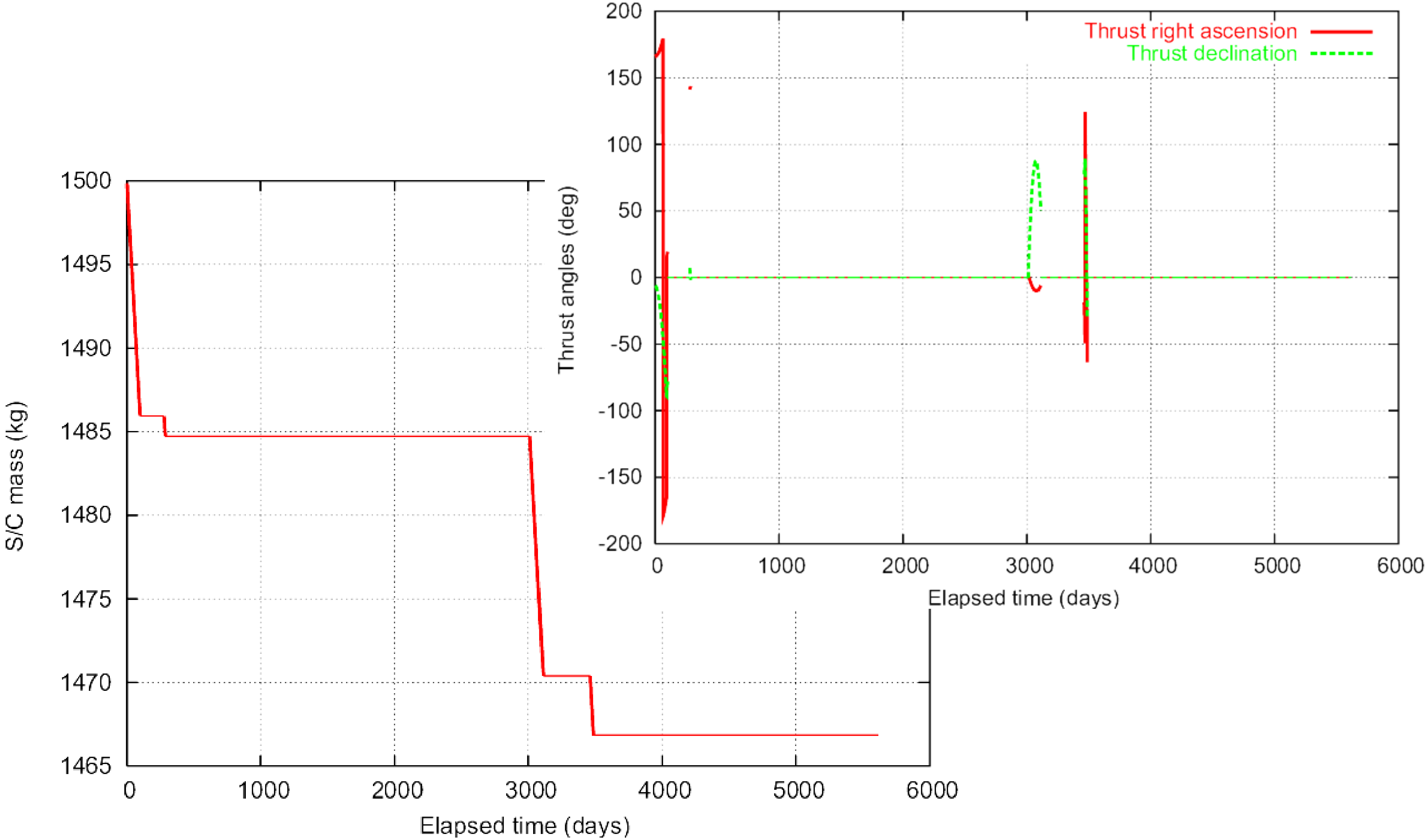
Obtained Solutions: CASE F



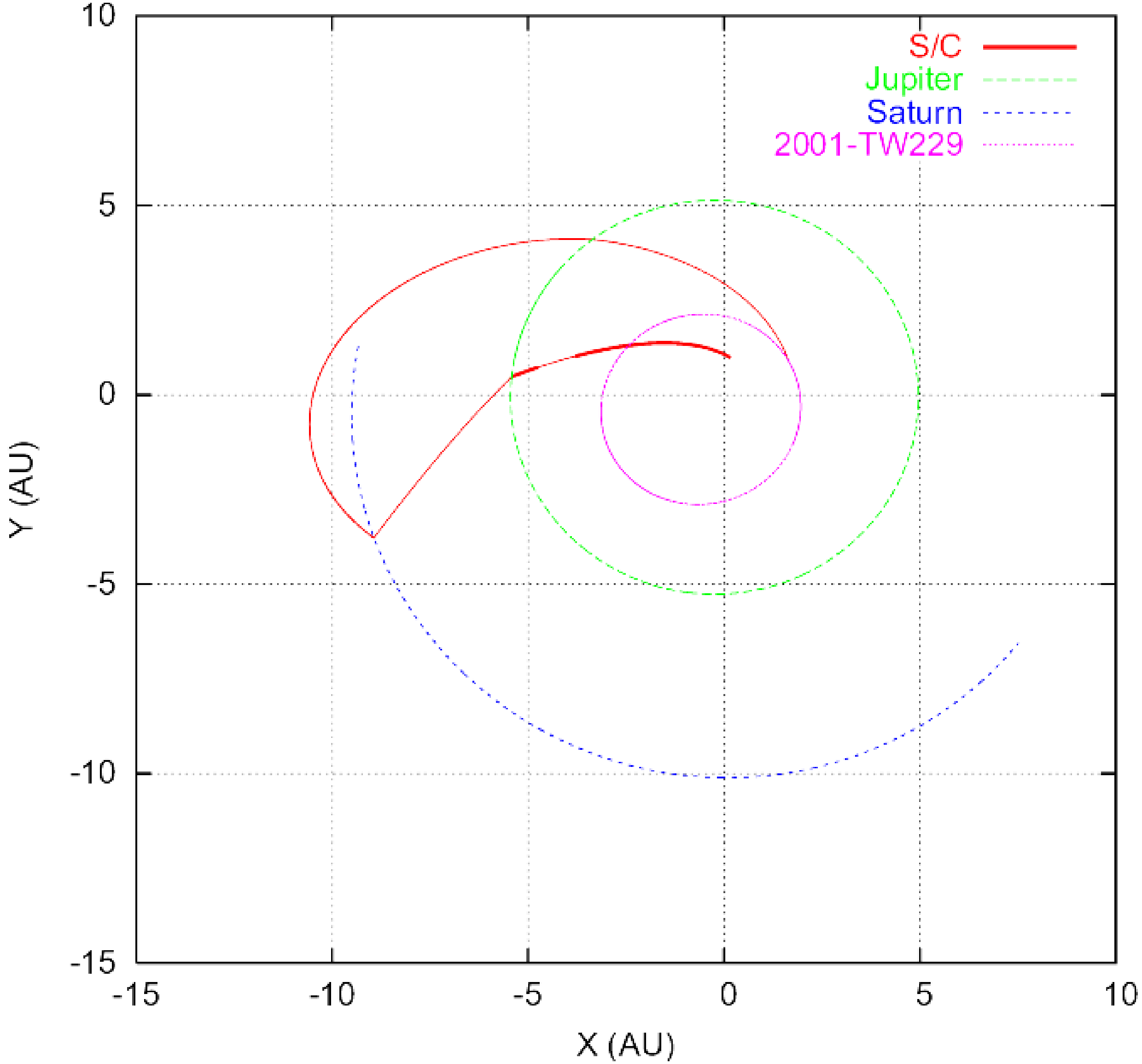
Obtained Solutions: CASE F



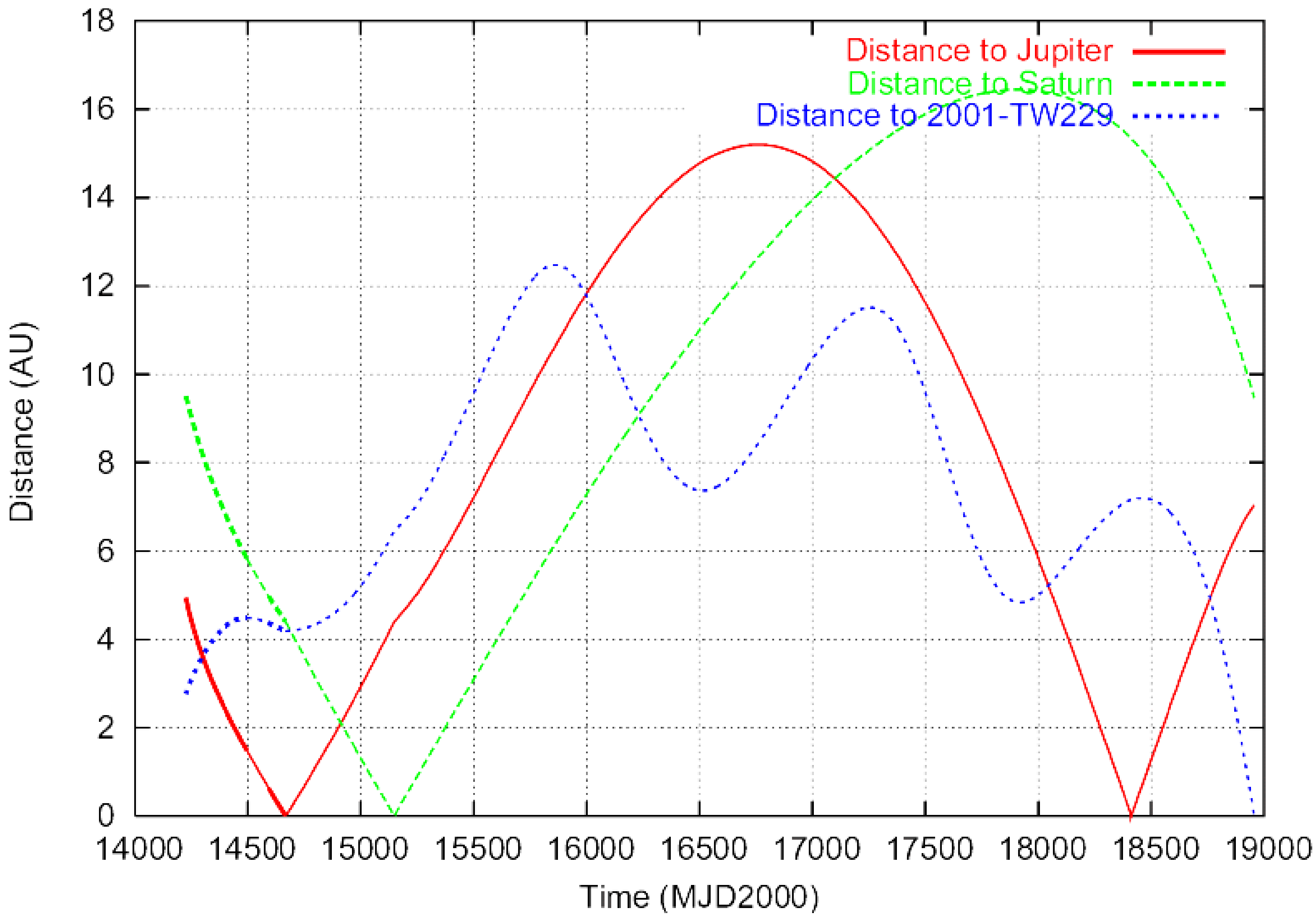
Obtained Solutions: CASE F



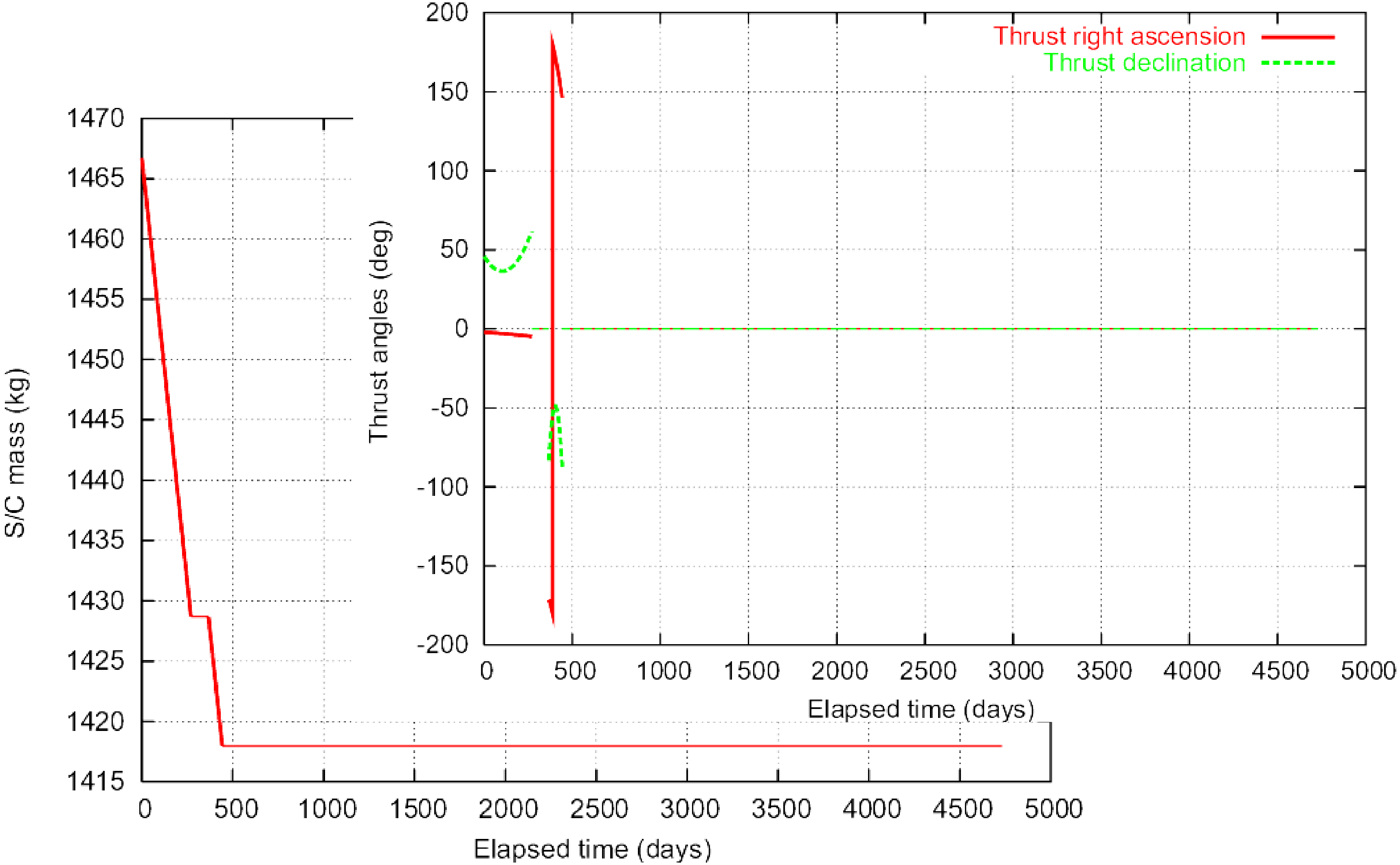
Obtained Solutions: CASE F



Obtained Solutions: CASE F



Obtained Solutions: CASE F



deimos SPACE

Obtained Solutions: CASE F

EVENT DATE	MAIN EVENT	THRUST SWITCH	ARCS	DURATION (days)
Departure	27/07/2023 02:28:54	ON	THRUST	99.6334
Thrust switch	03/11/2023 17:41:04	OFF	COAST	178.9369
Thrust switch	30/04/2024 16:10:09	ON	THRUST	8.7429
Thrust switch	09/05/2024 09:59:59	OFF	COAST	709.5614
Venus swingby	18/04/2026 23:28:28	---	COAST	786.4012
Venus swingby	13/06/2028 09:06:11	---	COAST	268.4548
Earth swingby	08/03/2029 20:01:04	---	COAST	365.2569
Earth swingby	09/03/2030 02:11:00	---	COAST	369.8145
Venus swingby	13/03/2031 21:43:53	---	COAST	224.6861
Venus swingby	24/10/2031 14:11:48	ON	THRUST	101.6304
Thrust switch	03/02/2032 05:19:37	OFF	COAST	350.2008
Earth swingby	18/01/2033 10:08:46	ON	THRUST	25.2106
Thrust switch	12/02/2033 15:11:58	OFF	COAST	1049.9888
Venus swingby	29/12/2035 14:55:47	---	COAST	1077.9008
Earth swingby	11/12/2038 12:32:53	ON	THRUST	270.6803
Thrust switch	08/09/2039 04:52:31	OFF	COAST	96.3831
Thrust switch	13/12/2039 14:04:15	ON	THRUST	76.1329
Jupiter swingby	27/02/2040 17:15:38	OFF	COAST	480.1008
Saturn swingby	21/06/2041 19:40:46	---	COAST	3264.543
Jupiter swingby	30/05/2050 8:43:00	---	COAST	543.769
Asteroid swingby	25/11/2051 3:10:10	---	END	---

Obtained Solutions: CASE F

	Epoch (MJD2000)	S/C Mass (Kg)	Arrival velocity X-Component (km/s)	Arrival velocity Y-Component (km/s)	Arrival velocity Z-Component (km/s)	Arrival velocity Modulus (km/s)	Flyby Radius (km)
DEPARTURE	8608.103	1500.00	-2.07121	-1.39994	0.01536	2.50000	---
VENUS	9604.978	1484.73	-2.59989	-1.72014	-3.76357	4.88700	15145.4
VENUS	10391.379	1484.73	-0.06323	-2.51365	4.17442	4.87321	10975.0
EARTH	10659.834	1484.73	7.72117	0.08648	-0.40024	7.73202	154408.8
EARTH	11025.091	1484.73	7.66688	-0.48858	-0.65798	7.71056	11055.5
VENUS	11394.906	1484.73	5.04346	12.43314	3.12243	13.77566	88852.1
VENUS	11619.592	1484.73	5.51672	12.23761	3.01919	13.75895	6351.0
EARTH	12071.423	1470.40	-4.79219	12.20541	5.28545	14.13765	16067.1
VENUS	13146.622	1466.85	-7.49181	6.31790	1.33294	9.89039	6351.0
EARTH	14224.523	1466.85	0.55171	14.72547	8.48581	17.00449	6678.0
JUPITER	14667.719	1414.04	-12.92630	6.02128	2.83227	14.53846	1160179.1
SATURN	15147.820	1444.27	-14.09410	-5.81645	-1.44151	15.31511	91422.7
JUPITER	18412.363	1444.27	24.19054	6.54639	1.97190	25.13813	10483338.1
ASTEROID	18956.132	1444.27	-11.98666	19.75972	8.09625	24.48828	---

Conclusions

- DEIMOS Space has provided a Mission Analysis study on the possibilities to impact with a large momentum the given asteroid and complying with the imposed constraints.
- A systematic approach to the attainment of the best solutions in a global perspective has been successfully put to practice with a very efficient and profound results.
- Full families of feasible solutions were obtained providing a thorough insight in the structure and casuistic of the solution space.
- A dedicated analysis has been performed on the most prominent cases, providing a full low-thrust optimisation.
- Solution E presents a cost function of 1.789510 · 106 kg · km2 / s2. The mission duration is 29.627 years.
- Solution F, presents a better cost function of 1.819872 · 106 kg · km2/s2. The mission duration is 28.331 years.
- Due to lack of time a solution with an even better cost function of 1.875 · 106 kg · km2/s2 and a mission duration of 28 years could not be converged before contest deadline