1st ACT Contest on Global Trajectory Optimisation
Solution Report

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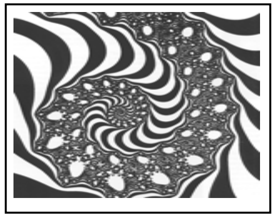
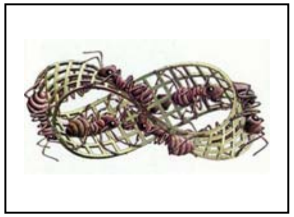
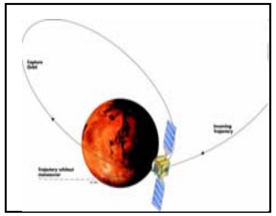
Team Composition



Problem Formulation



Approach, Strategies and Tools



The Solution

Final Remarks

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Outline

The team was made up of two principal groups one located in Glasgow (Scotland) and the other in Paderborn (Germany)

- ***University of Glasgow – UK***
  - *Massimiliano Vasile – Lecturer*
  - *Gianmarco Radice – Lecturer*
  - *Camilla Colombo – Research Assistant*
  - *Joan Pau Sanchez Cuartielles – Research Student*

- ***University of Strathclyde – UK***
  - *Colin McInnes – Professor of Engineering Sciences*

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Team Composition

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University of Paderborn – Germany

- *Michael Dellnitz – Full Professor*
- *Oliver Schütze – Post Doc*
- *Marcus Post – PhD Student*

- ***Munich University of Technology – Germany***
  - *Oliver Junge – Associate Professor*

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Team Composition

- Target asteroid 2001 TW229
- Launch window [3653-10958] MJD2000
- Maximum transfer time 30 years
- Launcher capability 2.5 km/sec
- No constraint on the escape asymptote
- Minimum allowed heliocentric distance of 0.2 AU
- Spacecraft initial mass 1500 kg
- Maximum thrust level 0.04N
- Isp=2500s
- Objective function:

$$\max \; J = m_f \left| \mathbf{U}_{rel} \cdot \mathbf{v}_{ast} \right|$$

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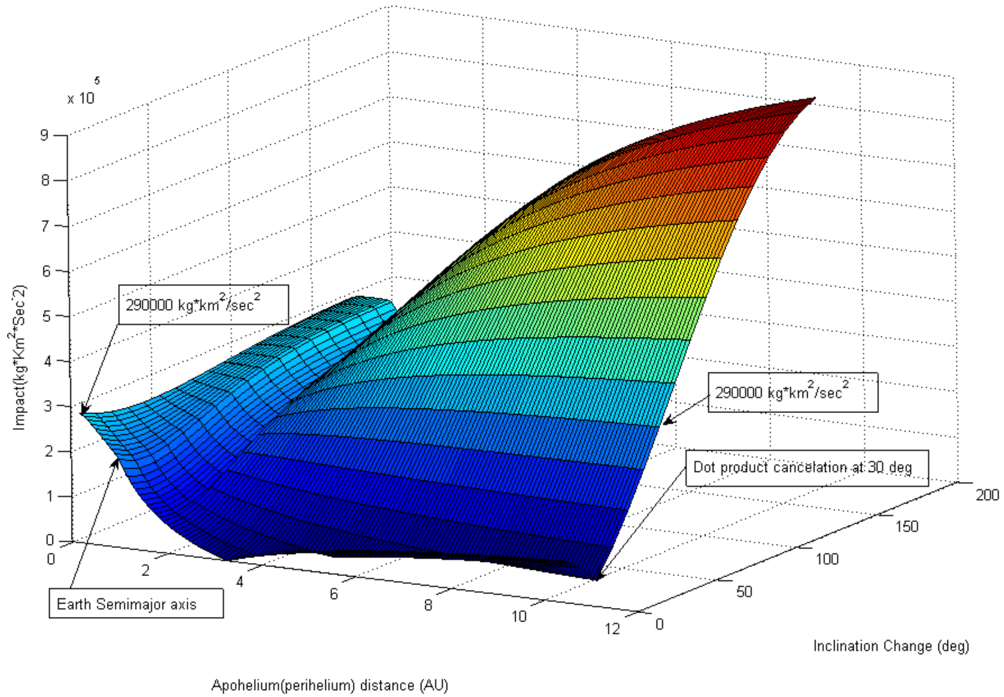
Problem Formulation



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Problem Formulation

Three different trajectory models

- Multiple gravity assist trajectory model with impulsive deep space manoeuvres
- Multiple gravity assist trajectory model with pseudo-equinoctial elements for low-thrust arcs description
- Multiple gravity assist trajectory model with exponential-sinusoid spirals for low-thrust arcs description

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1. Analytically propagate forward for a time t_i at celestial body p_i up to a point s_i
2. Compute a 2-impulse transfer, Lambert arc with transfer time T_i, from s_i to the next celestial body p_{i+1}
3. Solve exactly the GA manoeuvre as a function of the pericentre radius r_p and the hyperbola plane attitude angle ω
4. Add up all the Δv_i plus the initial Δv_0
5. Solve: $$\min_{\mathbf{x}\in D} f(\mathbf{x}) = \sum_{i=0}^{N} \Delta v_i$$

$$\mathbf{x} = \left[N, p_1, p_2, \ldots, p_N, t_0, \Delta v_0, t_1, T_1, \omega_1, r_{p1}, \ldots, t_i, T_i, \omega_i, r_{pi}, \ldots, t_N, T_N\right]$$

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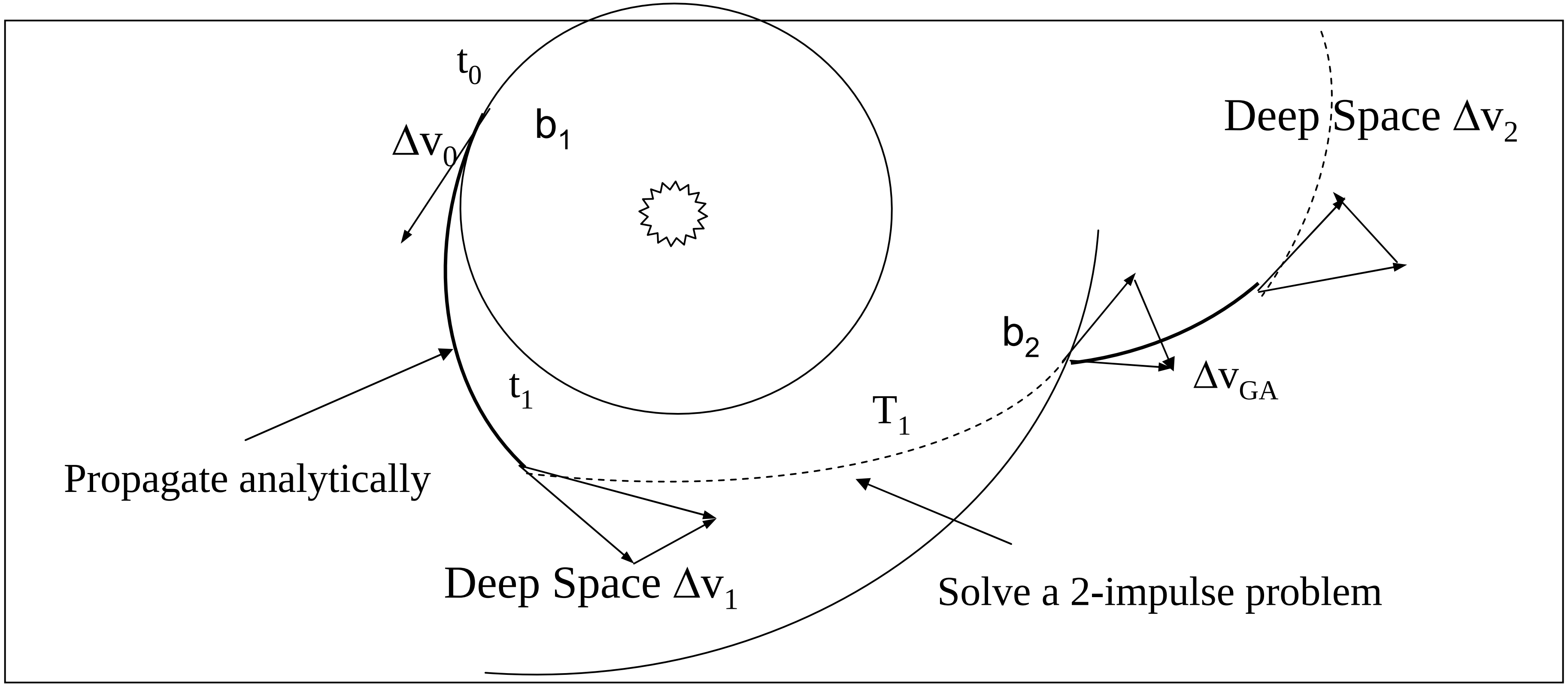
Tool Description: Trajectory Model



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Tool Description: Trajectory Model

- Trajectory expressed in terms of non-singular equinoctial elements:

$$\mathbf{r} = \mathbf{r}(p, f, g, h, k, L)$$

- Pseudo-equinoctial elements with exponential shape

$$\boldsymbol{\alpha} = \boldsymbol{\alpha}_0 + \delta\boldsymbol{\alpha} \quad \Rightarrow$$

$$\alpha_1 = \alpha_{01} + \alpha_{11} e^{\alpha_{21}(L-L_0)}$$

$$\alpha_2 = \alpha_{02} + \alpha_{12} e^{\alpha_{22}(L-L_0)}$$

$$\alpha_3 = \alpha_{03} + \alpha_{13} e^{\alpha_{22}(L-L_0)}$$

$$\alpha_4 = \alpha_{04} + \alpha_{14} e^{\alpha_{23}(L-L_0)}$$

$$\alpha_5 = \alpha_{05} + \alpha_{15} e^{\alpha_{23}(L-L_0)}$$

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Tool Description: Trajectory Model

- Inverse approach for the thrust profile

$$\mathbf{a}_d = \ddot{\mathbf{r}}(\boldsymbol{\alpha}(L)) + \mu \frac{\mathbf{r}(\boldsymbol{\alpha}(L))}{r(\boldsymbol{\alpha}(L))^3}$$

- where $\boldsymbol{\alpha} \equiv [p, f, g, h, k, -] \Rightarrow a_d = 0$
- Mass consumption is computed by numerical quadrature:

$$m_p = 1 - e^{-\int_{L_0}^{L_f} \frac{|\mathbf{a}_d|}{I_{sp} g_o} \frac{dt}{dL} dL}$$

- Terminal constraint on the TOF: $\Delta t = \int_{L_0}^{L_f} \frac{dt}{dL} dL$

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Tool Description: Trajectory Model

Exponential Sinusoid Model

- Given in 2D the following shape for the radius *r* as a function of the true anomaly θ:

$$r = k_0 \exp[k_1 \sin(k_2\theta + \phi)]$$

- The required control acceleration is :

$$a = \frac{\tan\gamma}{2\cos\gamma}\left[\frac{1}{\tan^2\gamma + k_1 k_2^2 s + 1} - \frac{k_2^2(1-2k_1 s)}{(\tan^2\gamma + k_1 k_2^2 s + 1)^2}\right]$$

- with:

$$\dot{\theta}^2 = \left(\frac{\mu}{r^3}\right)\frac{1}{\tan^2\gamma + k_1 k_2^2 s + 1}$$

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- By enforcing the following conditions at the boundaries:

$$\begin{cases} r_1 = k_0 \exp[k_1 \sin\phi] \\ r_2 = k_0 \exp[k_1 \sin(k_2\bar{\theta} + \phi)] \end{cases}$$

- With the additional condition on the initial flight path angle:

$$\tan\gamma_1 = k_1 k_2 \cos\phi$$

- It is possible to obtain k_0,k_1 and ϕ as a function of k_2 and γ_1

- No conditions on the extremal velocities can be enforced
- 3D powered GA manoeuvre for MGA trajectories

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Two different global optimisation tools have been used:

- EPIC at Glasgow University
- GAIO at Paderborn University

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- Reconstruct the set of feasible solutions **x** with property *P*

$$X = \{\mathbf{x} \in D \mid P(\mathbf{x})\}$$

- where

$$D = \{x_i \mid x_i \in [b_i^l, b_i^u] \subseteq \Re, i = 1, \ldots, n\}$$

- X converges to the global optimum set:

$$X_o = \{\mathbf{x}^* \in D \mid P(\mathbf{x}^*) \wedge \forall \mathbf{x} \in X \Rightarrow \mathbf{x}^* \;\mathrm{p}\; \mathbf{x}\}$$

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- The Multiagent Collaborative Search is a general framework for agent based stochastic methods, such as Particle Swarm or Ant Colonies.
- A population of agents, endowed with a set of behaviours, is deployed into the search space and seeks for solutions with the property *P*
- Some behaviours are peculiar to Evolutionary Algorithms others to PSO
- The MACS algorithm aims at reconstructing the optimal set X_0 or the closest suboptimal set *X*.
- Based on the output of the MACS step the solution space *D* is decomposed into subdomains D_l such that:

$$\bigcup_{l=1}^{M} D_l = D$$

- Some D_l are then pruned and others are explored further by MACS

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Global Optimization Using Dynamical Systems

Idea: Newton's method = discrete dynamical system

→ Use global dynamical systems numerics for global optimization

Software tool (Paderborn, Munich):

GAIO: “Global Analysis of Invariant Objects”

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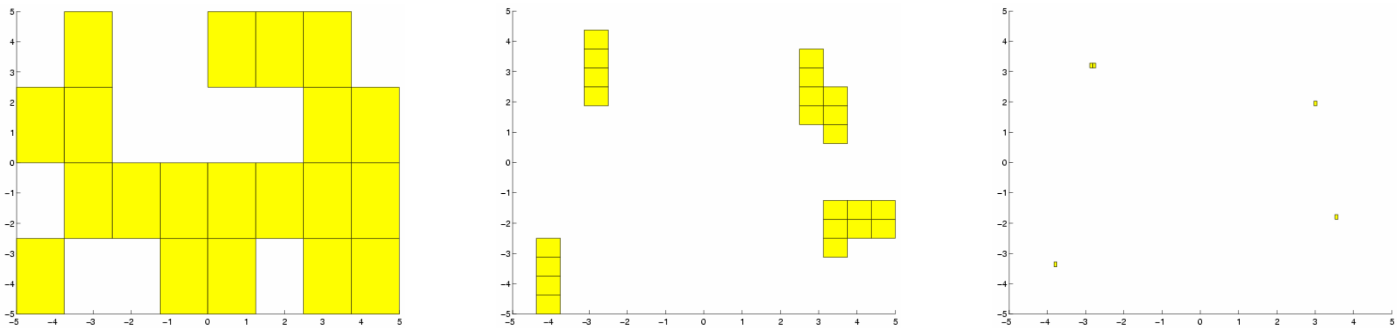
Tool Description: GAIO

Basic algorithm: subdivision scheme (“divide and prune”)



Pruning criterion based on dynamics of (e.g.) Newton's method

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Tool Description: GAIO

- The problem was tackled from different directions with different approaches.
- In particular the first attempt was to find optimal low-thrust direct trajectories.
- Two different shape-based methods were used to generate a first guess solution:
  - The exponential-sinusoid by Petropoulos et al.
  - The pseudo-equinoctial elements by De Pascale and Vasile

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Approach and Strategies

- Due to the expected higher reliability of the exponential-sinusoid the group in Paderborn initially used that trajectory model and GAIO as global optimiser
- The group in Glasgow used EPIC as global optimiser and the pseudo-equinoctial model for the trajectory
- The most promising first guesses were then re-optimised with the software DITAN

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- The result of this first round of optimisations was a number of trajectories with a value of the objective function not higher than 290e3 as first guess

- The exponential-sinusoid though generally produced trajectories with a high value of the thrust level

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- The second step was to study MGA trajectories. In particular two parallel directions were taken:
  - Inverting the motion through Jupiter or Saturn
  - Slow-down as much as possible with respect to the asteroid at the perihelion
- For the later strategy we used both shape-based methods to come up with a trajectory that could improve the value of the best direct one
- Unfortunately neither the exponential-sinusoid nor the pseudo-equinoctial one were yielding any trajectory with a feasible thrust level along the whole transfer

- To compute the motion-inversion trajectories we used the purely chemical model with impulsive manoeuvres

- Though all the trajectories going to Jupiter or Saturn were optimal and with a feasible escape velocity, the cost of the inversion manoeuvre was always excessive: higher than 10 km/s

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- The final step was to use the chemical model with impulsive manoeuvres to seek for optimal trajectories with multiple swingbys not inverting the motion

- Working in parallel we tried different combinations were tried: EVA, EEVA, EEVVA, EEVMMeA

- The most promising ones were then re-optimised with DITAN but with low-thrust manoeuvres instead of the chemical manoeuvres

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Parameter	Value	
Planetary sequence	E-E-V-V-TW229	
Launch Date	5 November 2027 (10170.5 MJD2000)	
Total ΔV (m/s)	5429.9	
Final mass (kg)	1202	
Earth Fly-by	1089.37 MJD2000 (t_0+723.17 d)	r_p=25176.5 km
Venus Fly-by	1107.27 MJD2000 (t_0+179 d)	r_p= 9179.4 km
Venus Fly-by	1148.61 MJD2000 (t_0+413.40 d)	r_p= 6351.8 km
Objective function (kg km^2/s^2)	385160	
Total transfer time (d)	2840.4	

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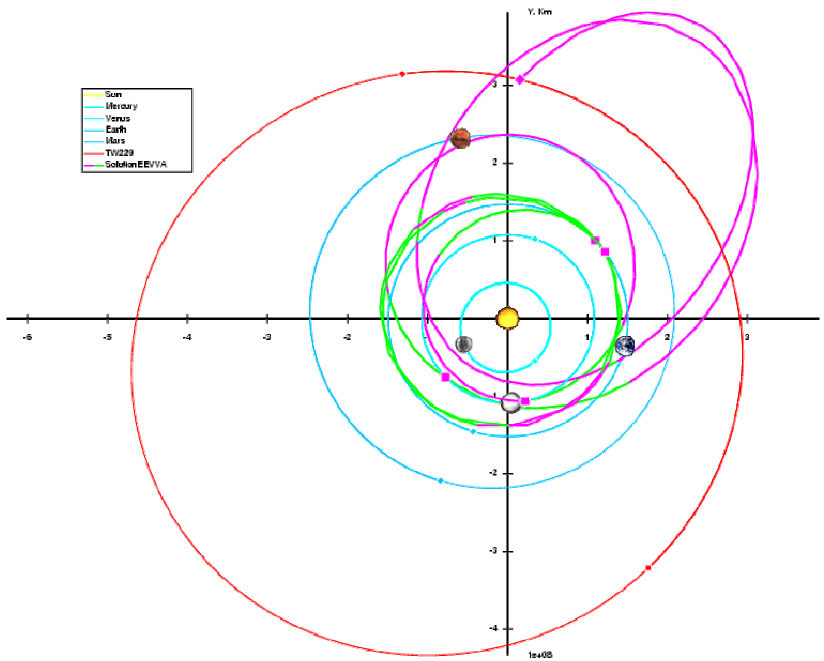

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• Optimal low-thrust trajectories are modified optimal impulsive trajectories

• Both shaping approaches overestimate mass consumption and thrust level

•Both global optimisation methods were quite effective providing a large number of solutions

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Final Remarks

Why couldn't we find an optimal motion-inversion trajectory?

The correct approach would have been to start with the chemical model since the beginning but…

…our Lambert solver was not computing retrograde orbits correctly!

So…

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Always remember to have a good Lambert solver with you!

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