ACT Global Optimization Competition Workshop

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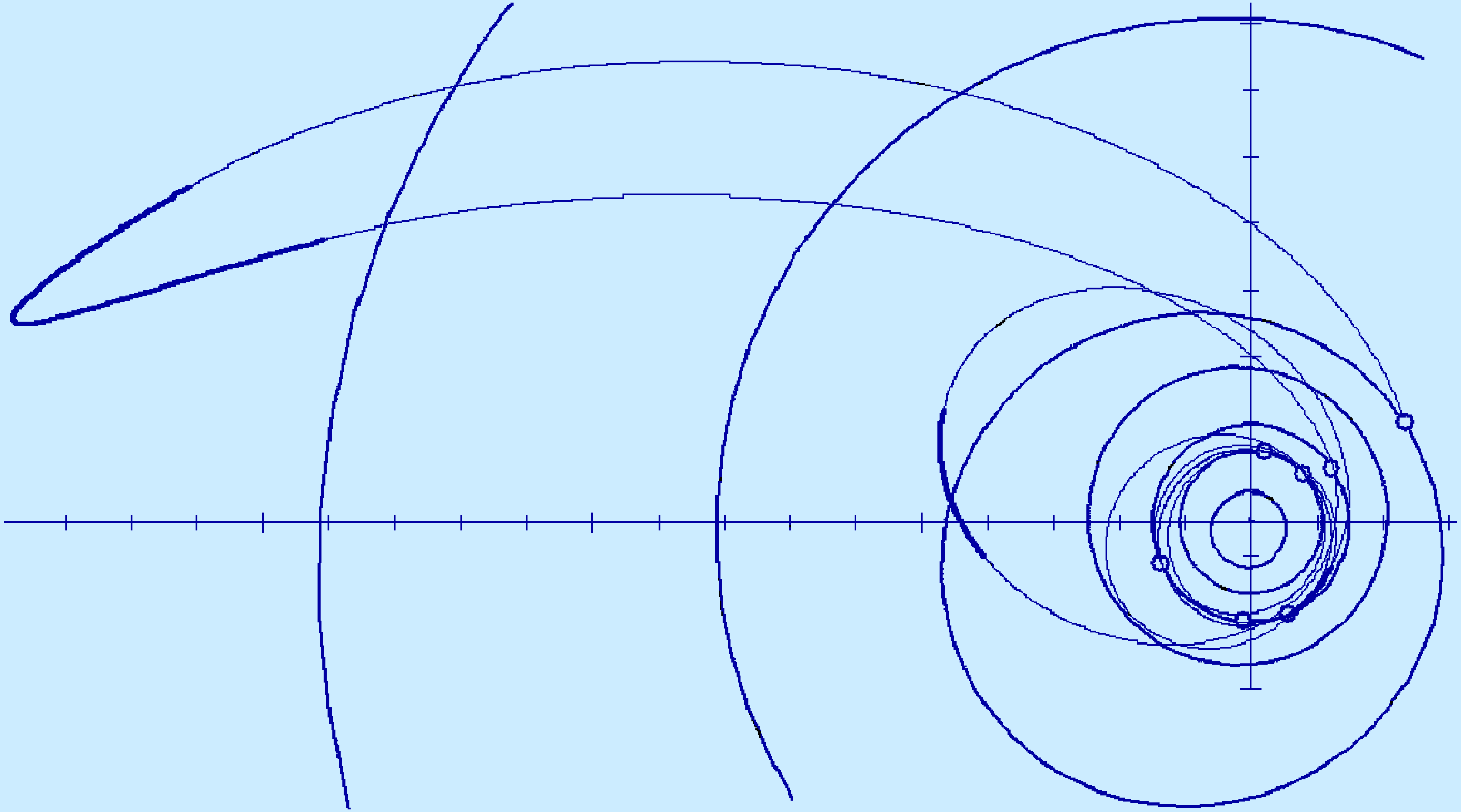


OBJECTIVE FUNCTION: PRELIMINARY CONSIDERATION

$$J = m_f \cdot \left|(\mathbf{v}-\mathbf{v}_{ast})^{\mathrm{T}}\mathbf{v}_{ast}\right|$$

Impact point in the vicinity of asteroid’s perihelion → Launch date and transfer duration should be optimized

1) Retrograde orbit
2) Impact in the vicinity of S/C perihelion
3) S/C aphelion distance should be maximized

→ Giant planets swingbys or low-thrust should be considered

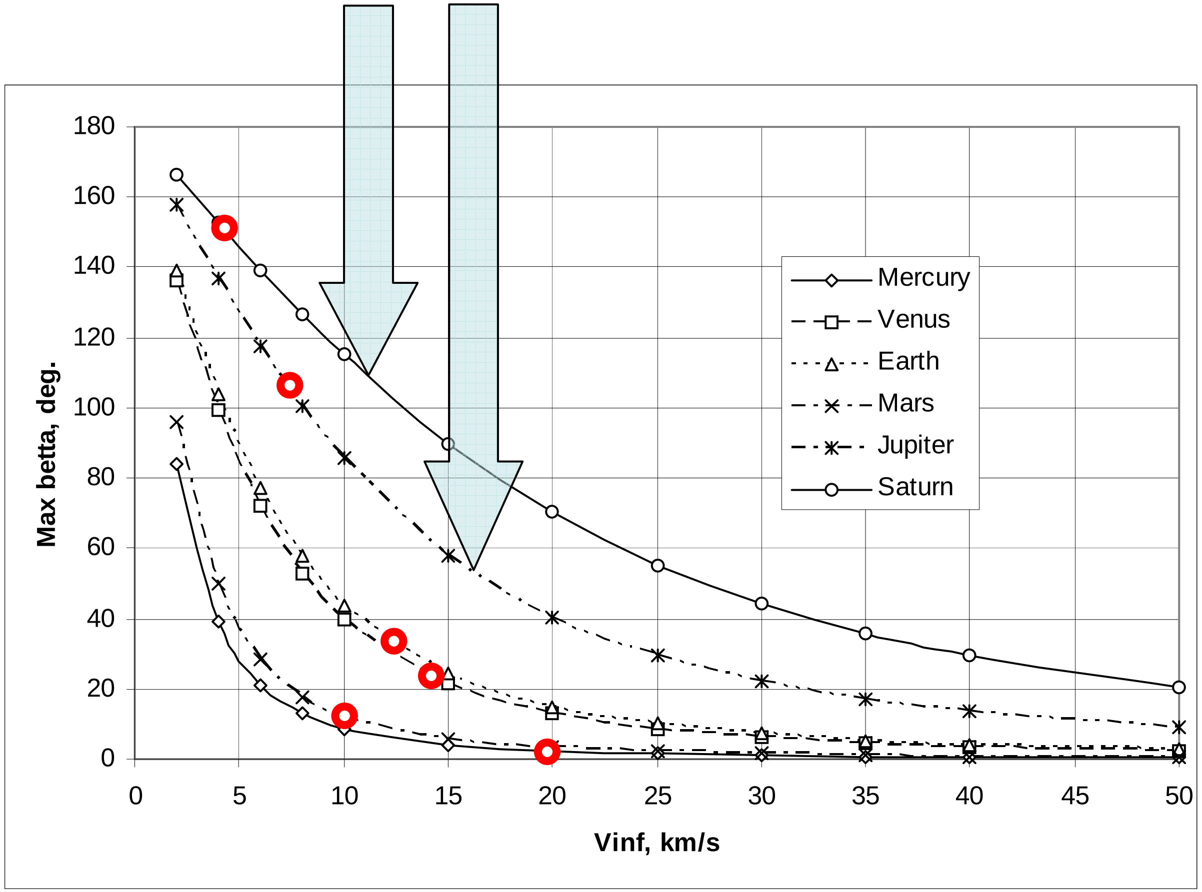
Minimum propellant →
1) Swingbys are prefered in comparison with thrusting
2) Transfer duration tends to maximal

THEREFORE...

Thanks ACT & Dario Izzo for an interesting problem

FLYBY CONSTRAINTS: ASYMPTOTIC VELOCITY ROTATION

Maximal angles ⇒ flyby sequences E…JSA or E…JSJA are prefered



- heliocentric parabola

LOW-THRUST TRAJECTORY: BOUNDARY CONDITIONS

Initial point:

$$\mathbf{x}(0) = \mathbf{x}_0,$$

$$\mathbf{v}(0) = \mathbf{v}_0 + V_\infty \frac{\mathbf{p}_\mathbf{v}(0)}{p_v(0)},$$

$$m(0) = m_0.$$

Final point:

$$\mathbf{x}(T) = \mathbf{x}_{ast},$$

$$\mathbf{p}_\mathbf{v}(T) = m(T)\,\mathrm{sgn}\left(\left(\mathbf{v}(T) - \mathbf{v}_{ast}\right)^\mathrm{T} \mathbf{v}_{ast}\right)\mathbf{v}_{ast},$$

$$p_m(T) = 1.$$

Swingby:

$$\mathbf{p}_\mathbf{v}^+ = \frac{\mathbf{V}_\infty^+}{V_\infty^-} p_v^-,$$

$$\mathbf{v}^+ = \mathbf{v}_{pl} + \mathbf{V}_\infty^+,$$

$$\mathbf{x} = \mathbf{x}_{pl},$$

where

$$\mathbf{V}_\infty^+ = \mathbf{M} \cdot \begin{pmatrix} \cos\beta \\ \sin\beta\cos\gamma \\ \sin\beta\sin\gamma \end{pmatrix}, \qquad \beta_{\max} = \pi - 2\arccos\left(\frac{1}{1 + \dfrac{r_{p\min} V_\infty^2}{\mu}}\right)$$

$$\mathbf{M} = \begin{pmatrix} V_{\infty x}^- & -\dfrac{V_{\infty y}^- V_\infty^-}{\sqrt{\left(V_{\infty x}^-\right)^2 + \left(V_{\infty y}^-\right)^2}} & -\dfrac{V_{\infty x}^- V_{\infty z}^-}{\sqrt{\left(V_{\infty x}^-\right)^2 + \left(V_{\infty y}^-\right)^2}} \\ V_{\infty y}^- & \dfrac{V_{\infty x}^- V_\infty^-}{\sqrt{\left(V_{\infty x}^-\right)^2 + \left(V_{\infty y}^-\right)^2}} & -\dfrac{V_{\infty y}^- V_{\infty z}^-}{\sqrt{\left(V_{\infty x}^-\right)^2 + \left(V_{\infty y}^-\right)^2}} \\ V_{\infty z}^- & 0 & \sqrt{\left(V_{\infty x}^-\right)^2 + \left(V_{\infty y}^-\right)^2} \end{pmatrix}$$

VERSIONS OF TRAJECTORY PROFILE AND COMPUTATIONAL TECHNIQUES

1. Flyby sequence in the inner Solar system (aphelion increasing + apsidal line positioning)

1.1. EV…E or EV…V coast flyby sequence

1.2. EE, EV, VE or VV trajectory using low-thrust arc

2. Retrograde trajectory shaping in the outer Solar system using Giants flybys and/or low thrust

EJSJA, EJSA, ESJA, ESA or EJA flyby sequence using low-thrust arcs if necessary

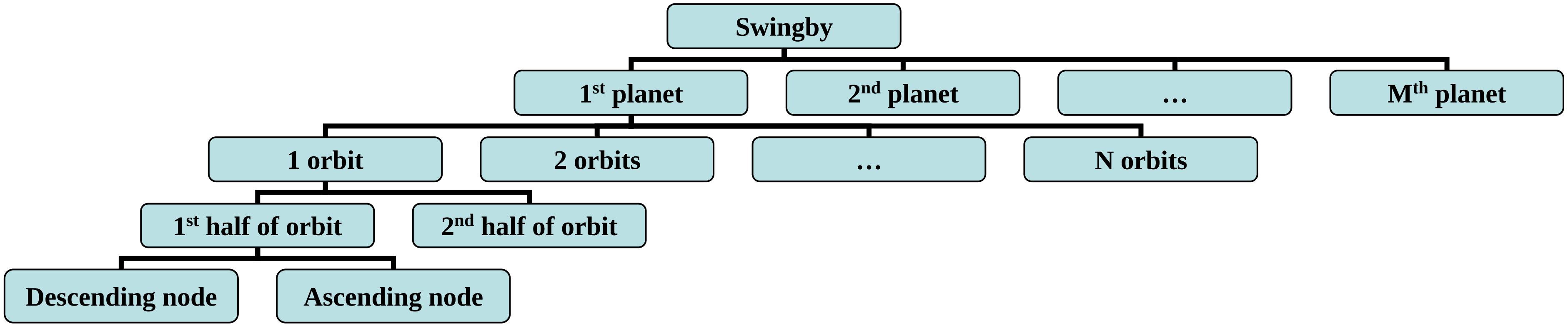
- or -

EA trajectory using low-thrust arc(s).

- Lambert solver
- Conic-patched interplanetary trajectory calculation based on the Kepler equation
- TPBVP solver (numerical integration, targeting to the next planet varying the trajectory arc duration and departure asymptotic velocity direction). Trajectory arc can include one thrusting arc using parametric thrust steering
- Power-limited (LP) problem solver
- Constant ejection velocity (LP-to-CEV) problem solver

PROBLEM COMPLEXITY

1. Large number of routes under consideration:



2. Local optimization of low-thrust arcs: bifurcations, numerical stability and convergence problems

Global optimization assumes using of reliable local optimization techniques.

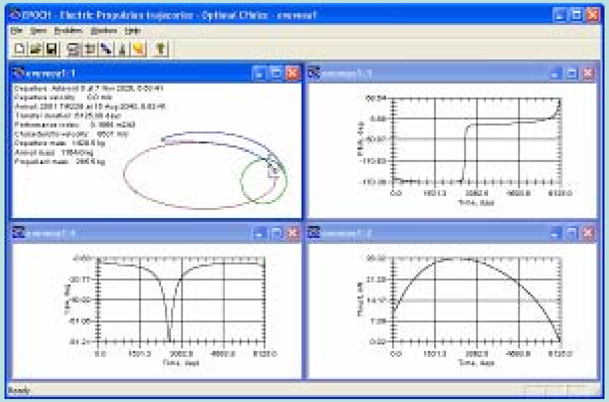
Local optimization technique of multiply swingbys low-thrust trajectory is insufficiently elaborated.

So, some simplifications are inevitable.

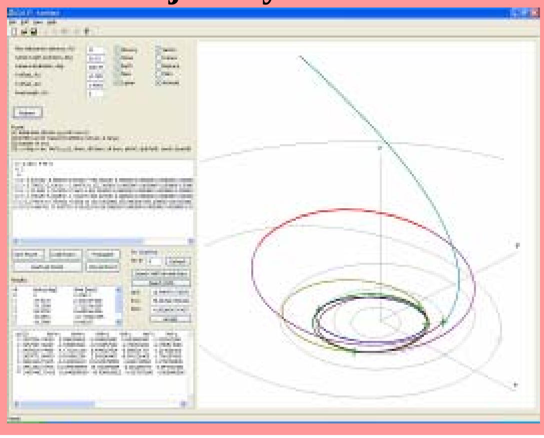
Due to problem complexity researcher's intuition and experience should be used along with optimization techniques using

SOFTWARE

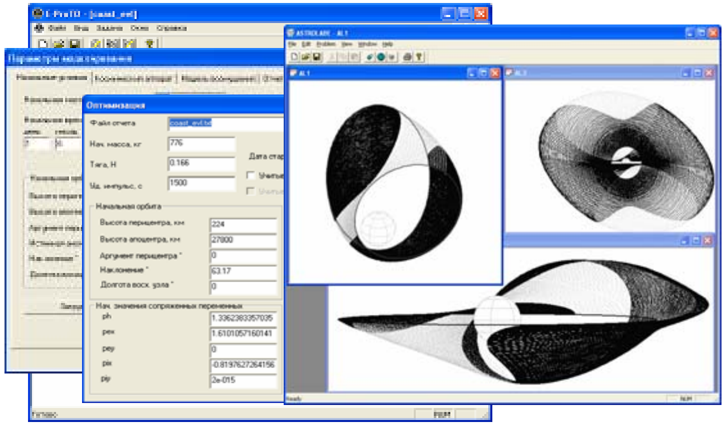
EPOCH – LP-problem solver



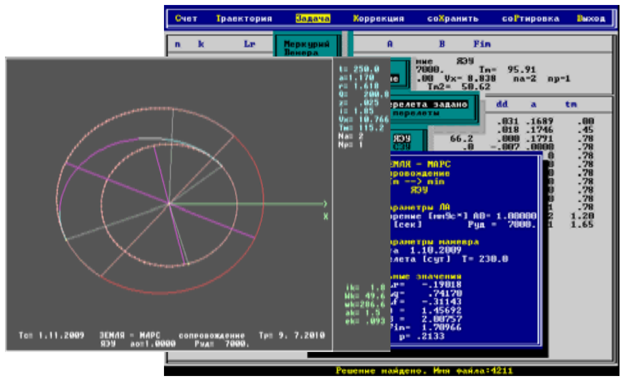
GALTT – Gravity-Assisted Low-Thrust Trajectory Evaluation



E-ProTO – multi-orbits trajectory optimization, CEV-problem



PL – CEV-problem solver



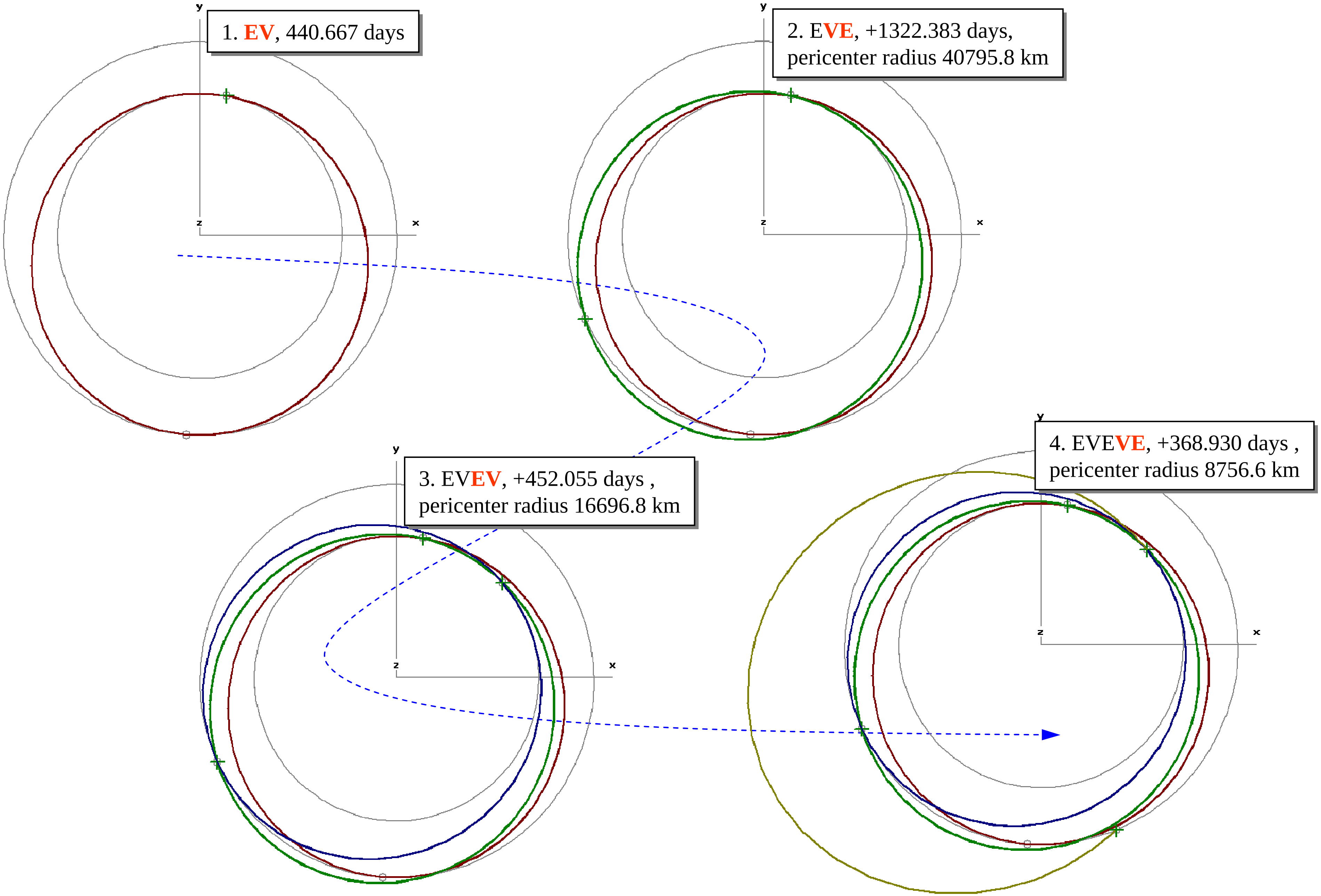
Other software:

- LP $\rightarrow$ CEV continuation (frozen trajectory structure)
- Lambert solvers
- Conic-patched interplanetary trajectory calculation based on the Kepler equation

Legend:

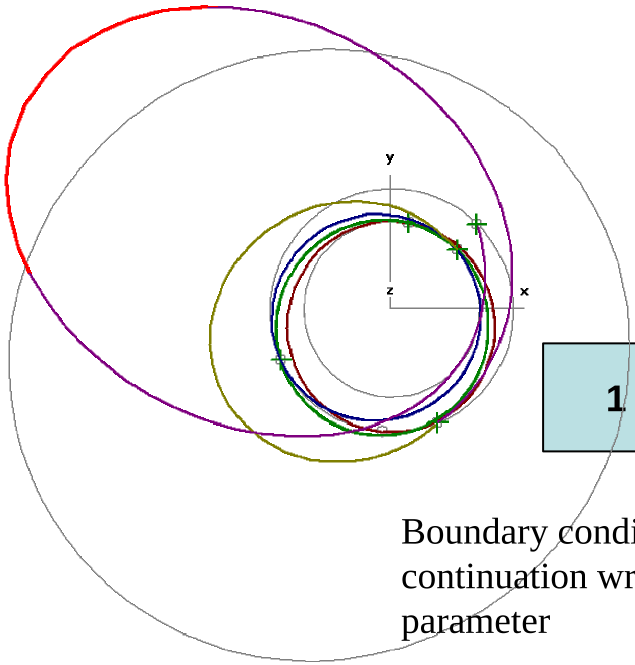
- (white) - previously developed and exploited software
- (light blue) - modified software
- (pink) - new-developed software

TRAJECTORY DESCRIPTION: INITIAL FLYBY SEQUENCE



TRAJECTORY DESCRIPTION: EARTH-to-EARTH TRAJECTORY ARC

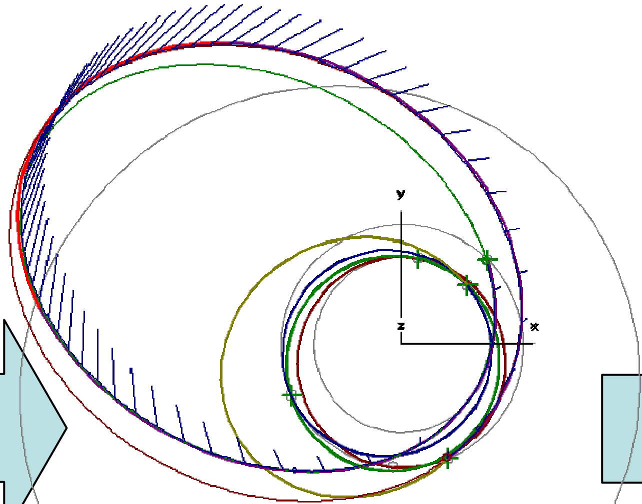
1. Tangential thrust (braking)



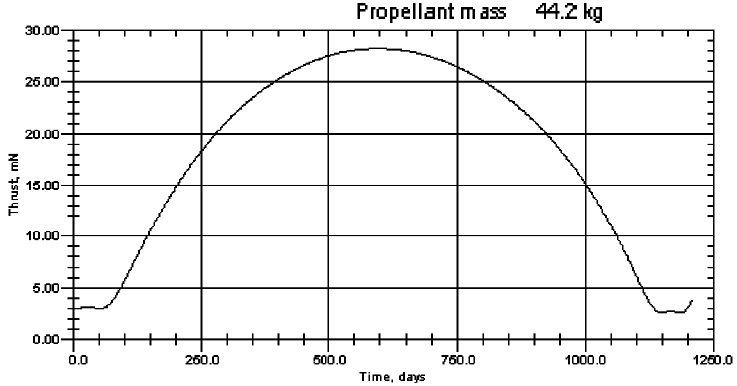
1

Boundary conditions, continuation wrt. gravity parameter

2. LP-trajectory



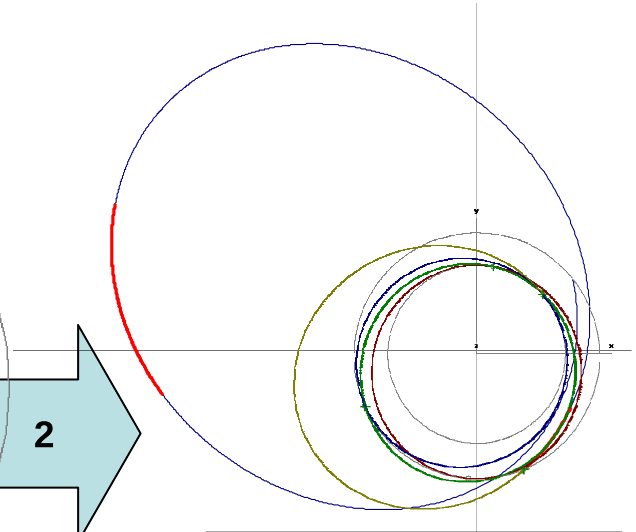
Departure: Asteroid 0 at 15 Jul 2023, 5:37:10
Departure velocity 0.0 m/s
Arrival: Ast2 at 7 Nov 2026, 5:53:40
Transfer duration: 1211.01 days
Performance index: 0.0198 m2/s3
Characteristic velocity: 1292 m/s
Departure mass 1500.0 kg
Arrival mass 1455.8 kg
Propellant mass 44.2 kg



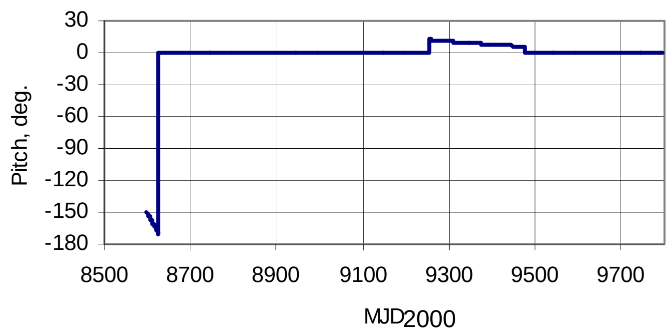
2

Continuation LP → CEV, transfer duration

3. CEV-trajectory



5. EVEVEE, +1200.0 days , pericenter radius 7936.0 km, thrusting 29.861 + 221.237 = 251.098 days



TYPICAL POWER-LIMITED PROBLEM

Purpose: To minimize $J = \frac{1}{2}\int_0^T \mathbf{a}^{\mathbf{T}}\mathbf{a}\,dt$ for a dynamical system obeying the differential equations $\frac{d^2\mathbf{x}}{dt^2} = \Omega_{\mathbf{x}} + \mathbf{a}$,
and having following boundary conditions: $t = 0$: $\mathbf{x}(0) = \mathbf{x}_0$, $d\mathbf{x}(0)/dt = \mathbf{v}_0$, $m(0) = m_0$, $t = T$: $\mathbf{x}(T) = \mathbf{x}_f$, $d\mathbf{x}(T)/dt = \mathbf{v}_f$.
Here T is final time, $\mathbf{a}$ is thrust acceleration, $\mathbf{x}$ and $\mathbf{v}$ are vectors of spacecraft position and velocity respectively, Ω is gravity potential.

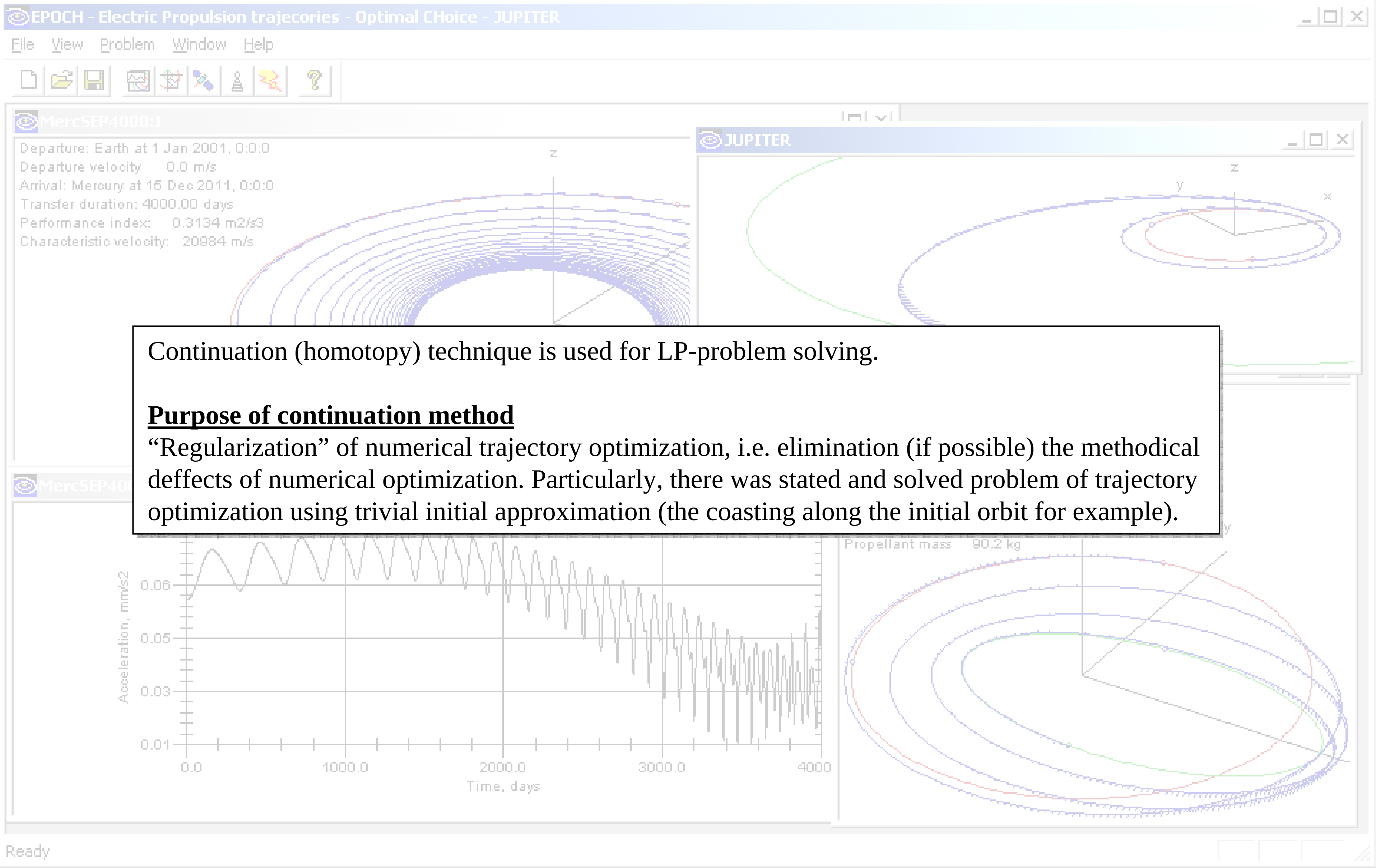
Let apply Pontryagin's maximum principle to reduce this OCP to the TPBVP.
Hamiltonian of this dynamical system is $H = -\mathbf{a}^{\mathbf{T}}\mathbf{a}/2 + \mathbf{p}_{\mathbf{x}}^{\mathbf{T}}\mathbf{v} + \mathbf{p}_{\mathbf{v}}^{\mathbf{T}}\Omega_{\mathbf{x}} + \mathbf{p}_{\mathbf{v}}^{\mathbf{T}}\mathbf{a}$. So, optimal control is $\mathbf{a} = \mathbf{p}_{\mathbf{v}}$ and equations of optimal motion become following:

$$\left.\begin{aligned} \frac{d^2\mathbf{x}}{dt^2} &= \Omega_{\mathbf{x}} + \mathbf{p}_{\mathbf{v}}, \\ \frac{d^2\mathbf{p}_{\mathbf{v}}}{dt^2} &= -\Omega_{\mathbf{xx}}\mathbf{p}_{\mathbf{v}}, \end{aligned}\right\}$$

The boundary conditions for rendezvous mission has form: $t = 0$: $\mathbf{x}(0) = \mathbf{x}_0$, $d\mathbf{x}(0)/dt = \mathbf{v}_0$, $t = T$: $\mathbf{x}(T) = \mathbf{x}_f$, $d\mathbf{x}(T)/dt = \mathbf{v}_f$.

In fact, it is necessary to solve equation $\mathbf{f}(\mathbf{z}) = 0$, where $\mathbf{f}(\mathbf{z}) = \begin{pmatrix} \mathbf{x}(T) - \mathbf{x}_f \\ \mathbf{v}(T) - \mathbf{v}_f \end{pmatrix}$ is vector of residuals, $\mathbf{z} = \begin{pmatrix} \mathbf{p}_{\mathbf{x}}(0) \\ \mathbf{p}_{\mathbf{v}}(0) \end{pmatrix}$ is vector of unknown TPBVP parameters, $\mathbf{p}_{\mathbf{x}} = -d\mathbf{p}_{\mathbf{v}}/dt$.

ALGORITHMS FOR POWER-LIMITED PROBLEM



Continuation (homotopy) technique is used for LP-problem solving.

Purpose of continuation method

“Regularization” of numerical trajectory optimization, i.e. elimination (if possible) the methodical deffects of numerical optimization. Particularly, there was stated and solved problem of trajectory optimization using trivial initial approximation (the coasting along the initial orbit for example).

CONTINUATION TECHNUQUE

Continuation (Homotopy) Technique

Problem: to solve non-linear system

$$\boxed{\mathbf{f}(\mathbf{z}) = 0} \qquad (1)$$

with respect to vector $\mathbf{z}$

Let $\mathbf{z}_0$ - initial approximation of solution. Then

$$\mathbf{f}(\mathbf{z_0}) = \mathbf{b} \qquad (2)$$

where $\mathbf{b}$ - residuals when $\mathbf{z} = \mathbf{z}_0$.

Let consider immersion $\mathbf{z}(\tau)$, where τ is a scalar parameter, and equation

$$\mathbf{f}(\mathbf{z}) = (1-\tau)\mathbf{b} \qquad (3)$$

with respect to $\mathbf{z}(\tau)$. Obviously, $\mathbf{z}(1)$ is solution of eq. (1).
Let differentiate eq. (2) on τ and resolve it with respect to dz/dτ:

$$\boxed{\frac{d\mathbf{z}}{d\tau} = -\mathbf{f}_{\mathbf{z}}^{-1}(\mathbf{z})\mathbf{b}, \quad \mathbf{z}(0) = \mathbf{z_0}} \qquad (4)$$

Just after integrating eq. (4) from 0 to 1 we have solution of eq. (1).

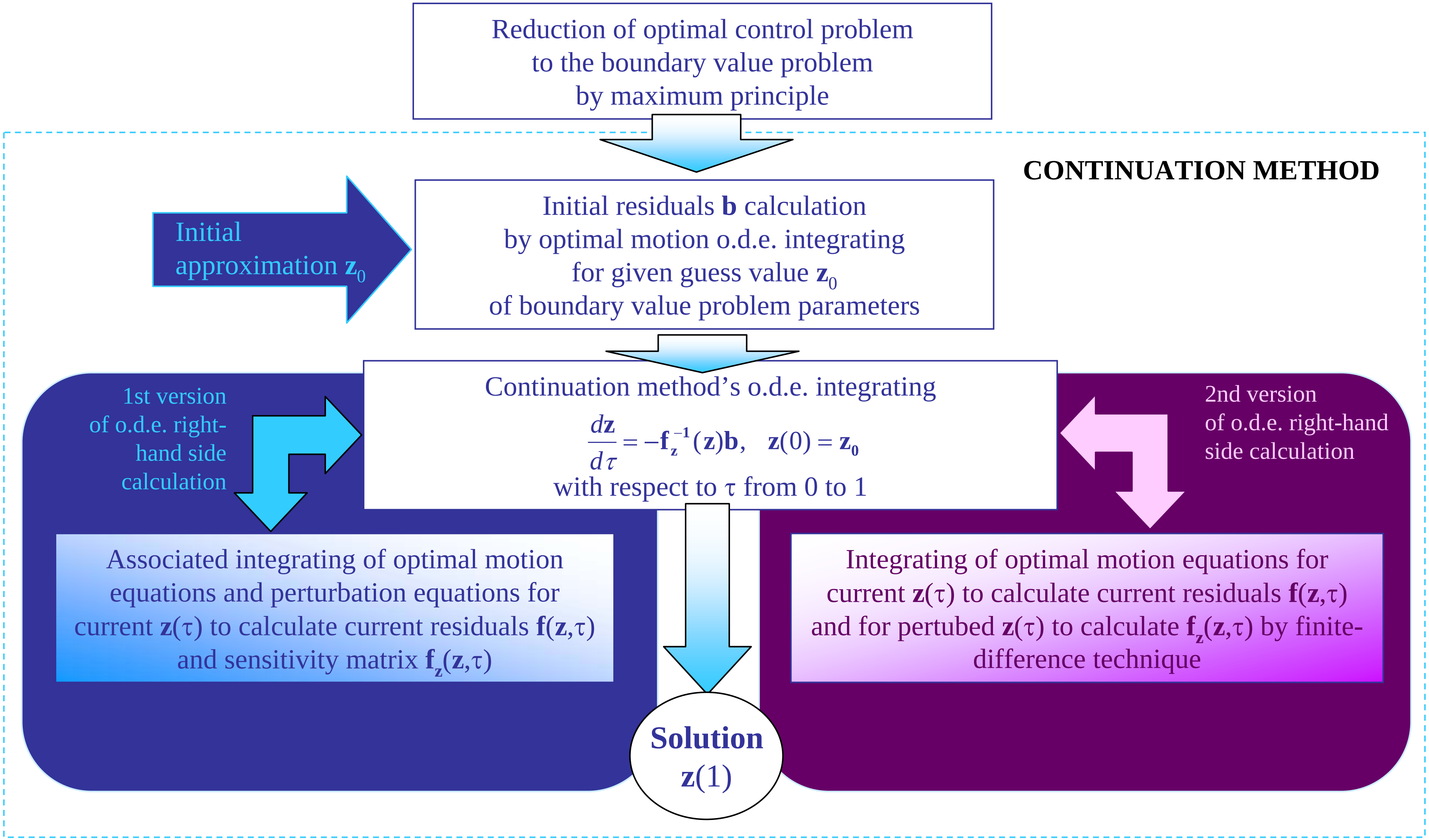
Equation (4) is the differential equation of continuation algorithm (the formal reduction of non-linear system (1) into initial value problem (4)).

Differential equations (4) are integrated using a high-order Runge-Kutta method with adaptive step size control.

Application to Optimal Control Problem (OCP)

- Optimal motion equations (after applying the maximum principle): $$\left.\begin{aligned} \frac{d\mathbf{x}}{dt} &= H_{\mathbf{p}}, \\ \frac{d\mathbf{p}}{dt} &= -H_{\mathbf{x}} \end{aligned}\right\}$$
- Boundary conditions (an example): $\mathbf{x}(0) = \mathbf{x}_0, \mathbf{x}(T) = \mathbf{x}_k$
- Boundary value problem parameters and residuals: $\mathbf{z} = \mathbf{p}(0), \mathbf{f} = \mathbf{x}(T) - \mathbf{x}_k$
- Sensitivity matrix: $\mathbf{f}_{\mathbf{z}} = \dfrac{\partial \mathbf{x}(T)}{\partial \mathbf{z}}$
- Associated system of optimal motion o.d.e. and perturbation equations for residuals and sensitivity matrix calculation: $$\left.\begin{aligned} \frac{d\mathbf{x}}{dt} &= H_{\mathbf{p}}, \\ \frac{d\mathbf{p}}{dt} &= -H_{\mathbf{x}}, \\ \frac{d}{dt}\left(\frac{\partial \mathbf{x}}{\partial \mathbf{z}}\right) &= H_{\mathbf{px}}\frac{\partial \mathbf{x}}{\partial \mathbf{z}} + H_{\mathbf{pp}}\frac{\partial \mathbf{p}}{\partial \mathbf{z}}, \\ \frac{d}{dt}\left(\frac{\partial \mathbf{p}}{\partial \mathbf{z}}\right) &= -H_{\mathbf{xx}}\frac{\partial \mathbf{x}}{\partial \mathbf{z}} - H_{\mathbf{xp}}\frac{\partial \mathbf{p}}{\partial \mathbf{z}} \end{aligned}\right\}$$
- Extended initial conditions: $\mathbf{x}(0) = \mathbf{x}_0, \mathbf{x}(T) = \mathbf{x}_k, \dfrac{\partial \mathbf{x}}{\partial \mathbf{z}} = 0, \dfrac{\partial \mathbf{p}}{\partial \mathbf{z}} = \mathbf{I}$

OCP-SOLVER BASED ON CONTINUATION ALGORITHM



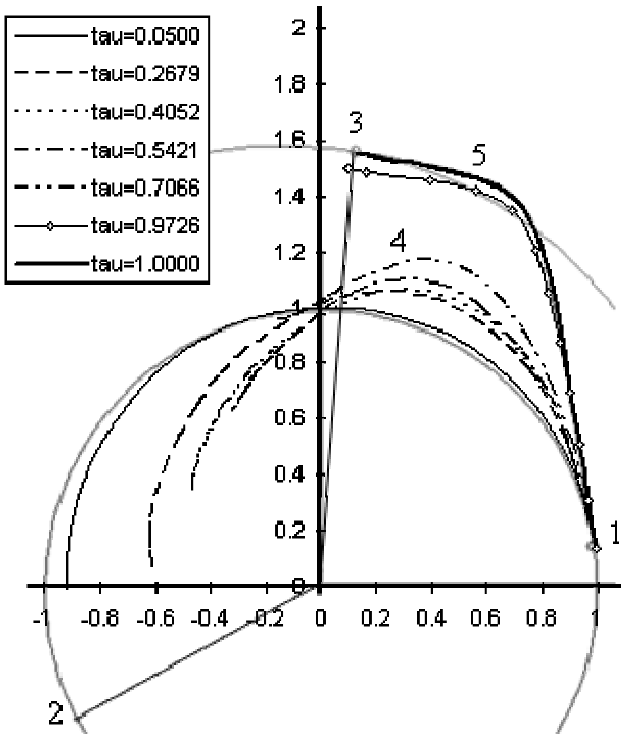
CONTINUATION WITH RESPECT TO GRAVITY PARAMETER (1/3)

<u>Occasional reasons of continuation algorithm failure:</u> sensitivity matrix degeneration (bifurcation of optimal solutions)

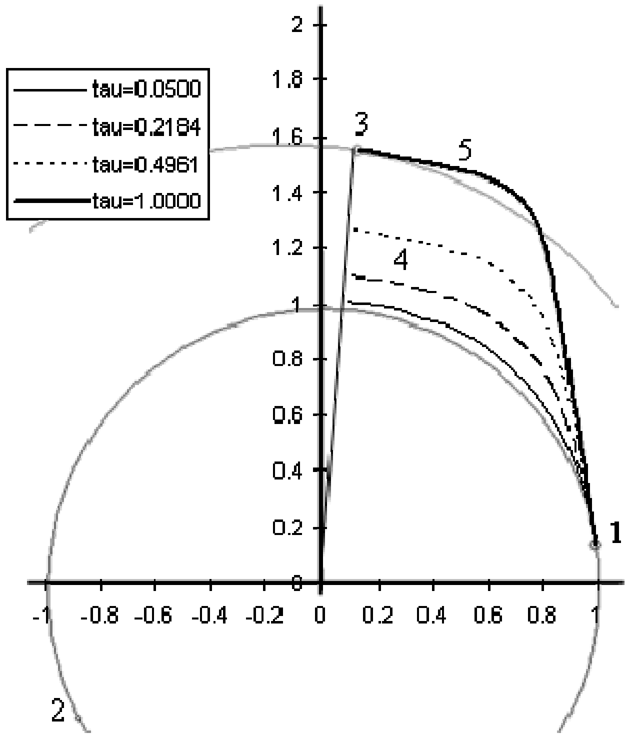
Mostly bifurcations of optimal planetary trajectories are connected with different number of complete orbits.

→ If angular distance will remain constant during continuation, the continuation way in the parametric space will not cross boundaries of different kinds of optimal trajectories. So, the sensitivity matrix will not degenerate.

The purpose of the technique modification - to fix angular distance of transfer during continuation



Sequence of trajectory calculations using basic continuation method



Sequence of trajectory calculations using continuation with respect to gravity parameter

Earth-to-Mars, rendezvous,
launch date October 1, 2001,
V_∞= 0 m/s, T=200 days

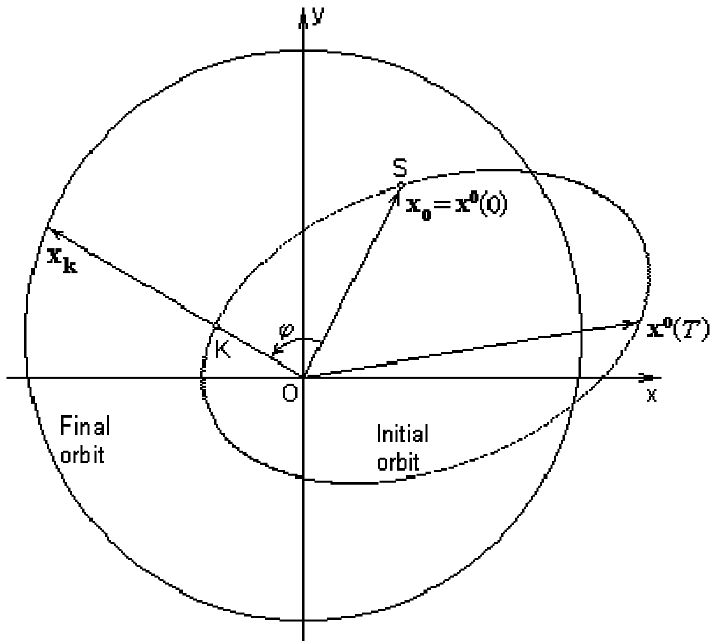
1 - Earth at launch
2 - Earth at arrival
3 - Mars at arrival
4 - intermediate trajectories (τ < 1)
5 - optimal trajectory (τ = 1)

CONTINUATION WITH RESPECT TO GRAVITY PARAMETER (2/3)

Let $\mathbf{x}_0(0)$, $\mathbf{x}_0(T)$ - departure planet position when t=0 and $t=T$;
$\mathbf{x}_k$ - target planet position when $t=T$. <u>Let suppose primary gravity parameter to be linear function of τ</u>, and let choose initial value of this gravity parameter μ_0 using following condition:

1) angular distances of transfer are equal when τ=0 and τ=1;
2) When τ=1 primary gravity parameter equals to its real value (1 for dimensionless equations)

The initial approximation is SC coast motion along departure planet orbit. Let the initial true anomaly equals to ν_0 at the start point S, and the final one equals to $\nu_k=\nu_0+\varphi$ at the final point K (φ is angle between $\mathbf{x_0}$ and projection of $\mathbf{x_k}$ into the initial orbit plane).



The solution of Kepler equation gives corresponding values of mean anomalies M_0 and M_k ($M=E-e\cdot\sin E$, where $E=2\cdot\mathrm{arctg}\{[(1-e)/(1+e)]^{0.5}\mathrm{tg}(\nu/2)\}$ is eccentric anomaly). Mean anomaly is linear function of time at the keplerian orbit: $M=M_0+n\cdot(t-t_0)$, where $n=(\mu_0/a^3)^{0.5}$ is mean motion. Therefore, the condition of angular distance invariance is $M_k+2\pi\, N_{rev}=nT+M_0$, where N_{rev} is number of complete orbits. So initial value of the primary gravity parameter is

$$\mu_0=[(M_k+2\pi\, N_{rev} - M_0)/T]^2a^3,$$

and current one is

$$\mu(\tau)=\mu_0+(1-\mu_0)\,\tau.$$

The shape and size of orbits should be invariance with respect to τ, therefore

$$\mathbf{v}(t,\tau)=\mu(\tau)^{0.5}\,\mathbf{v}(t,1).$$

CONTINUATION WITH RESPECT TO GRAVITY PARAMETER (3/3)

Equations of motion:

$$\ddot{\mathbf{x}} = \mu(\tau)\Omega_{\mathbf{x}} + \mathbf{p}_{\mathbf{v}}, \quad \ddot{\mathbf{p}}_{\mathbf{v}} = \mu(\tau)\Omega_{\mathbf{xx}}\mathbf{p}_{\mathbf{v}}$$

Boundary conditions:

$$\mathbf{x}(0) = \mathbf{x}_{\mathbf{0}}, \quad \dot{\mathbf{x}}(0) = \mu^{1/2}(\tau)\mathbf{v}_{\mathbf{0}},$$
$$\mathbf{x}(T) = \mathbf{x}_{\mathbf{k}}, \quad \dot{\mathbf{x}}(T) = \mu^{1/2}(\tau)\mathbf{v}_{\mathbf{k}}.$$

Residuals:

$$\mathbf{f} = \begin{pmatrix} \mathbf{x}(T) - \mathbf{x}_{\mathbf{k}} \\ \dot{\mathbf{x}}(T) - \mu^{1/2}(\tau)\mathbf{v}_{\mathbf{k}} \end{pmatrix}$$

Boundary value problem parameters:

$$\mathbf{z} = (\mathbf{p}_{\mathbf{v}}(0), d\mathbf{p}_{\mathbf{v}}(0)/dt)^{T} = \left(\mathbf{p}_{\mathrm{vo}}, \dot{\mathbf{p}}_{\mathrm{vo}}\right)^{T}$$

Equation of continuation method:

$$\frac{d\mathbf{z}}{d\tau} = -\mathbf{f}_{\mathbf{z}}^{-1}(\mathbf{z})\left(\mathbf{b} + \frac{\partial \mathbf{f}}{\partial \tau}\right), \quad \mathbf{z}(0) = \mathbf{z}_{\mathbf{0}}$$

where

$\mathbf{b} = \mathbf{f}(\mathbf{z}_0)$

$$\mathbf{f}_{\mathbf{z}} = \begin{pmatrix} \partial \mathbf{x}(T)/\partial \mathbf{p}_{\mathrm{vo}} & \partial \mathbf{x}(T)/\partial \dot{\mathbf{p}}_{\mathrm{vo}} \\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\partial \mathbf{x}(T)/\partial \mathbf{p}_{\mathrm{vo}}\right) & \frac{\mathrm{d}}{\mathrm{d}t}\left(\partial \mathbf{x}(T)/\partial \dot{\mathbf{p}}_{\mathrm{vo}}\right) \end{pmatrix}$$

$$\frac{\partial \mathbf{f}}{\partial \tau} = \begin{pmatrix} \frac{\partial \mathbf{x}}{\partial \tau} \\ \frac{\partial \dot{\mathbf{x}}}{\partial \tau} - \frac{1}{2\mu^{1/2}(\tau)}\frac{\partial \mu}{\partial \tau}\mathbf{v}_{\mathbf{k}} \end{pmatrix}$$

$$\left.\begin{aligned}
&\frac{d^2}{dt^2}\left(\frac{\partial \mathbf{x}}{\partial \mathbf{z}}\right) = \mu(\tau)\Omega_{\mathbf{xx}}\frac{\partial \mathbf{x}}{\partial \mathbf{z}} + \frac{\partial \mathbf{p}_{\mathbf{v}}}{\partial \mathbf{z}}, \\
&\frac{d^2}{dt^2}\left(\frac{\partial \mathbf{p}_{\mathbf{v}}}{\partial \mathbf{z}}\right) = \mu(\tau)\left[\frac{\partial}{\partial \mathbf{x}}\left(\Omega_{\mathbf{xx}}\mathbf{p}_{\mathbf{v}}\right)\frac{\partial \mathbf{x}}{\partial \mathbf{z}} + \Omega_{\mathbf{xx}}\frac{\partial \mathbf{p}_{\mathbf{v}}}{\partial \mathbf{z}}\right], \\
&\frac{d^2}{dt^2}\left(\frac{\partial \mathbf{x}}{\partial \tau}\right) = \frac{\partial \mu}{\partial \tau}\Omega_{\mathbf{x}} + \mu(\tau)\Omega_{\mathbf{xx}}\frac{\partial \mathbf{x}}{\partial \mathbf{z}} + \frac{\partial \mathbf{p}_{\mathbf{v}}}{\partial \mathbf{z}}, \\
&\frac{d^2}{dt^2}\left(\frac{\partial \mathbf{p}_{\mathbf{v}}}{\partial \tau}\right) = \frac{\partial \mu}{\partial \tau}\Omega_{\mathbf{xx}}\mathbf{p}_{\mathbf{v}} + \mu(\tau)\left[\frac{\partial}{\partial \mathbf{x}}\left(\Omega_{\mathbf{xx}}\mathbf{p}_{\mathbf{v}}\right)\frac{\partial \mathbf{x}}{\partial \mathbf{z}} + \Omega_{\mathbf{xx}}\frac{\partial \mathbf{p}_{\mathbf{v}}}{\partial \mathbf{z}}\right],
\end{aligned}\right\}$$

$$\frac{\partial \mathbf{x}(0)}{\partial \mathbf{z}} = \frac{\partial \dot{\mathbf{x}}(0)}{\partial \mathbf{z}} = \frac{\partial \mathbf{x}(0)}{\partial \tau} = 0, \quad \frac{\partial \dot{\mathbf{x}}(0)}{\partial \tau} = \frac{1}{2\mu^{1/2}(\tau)}\frac{\partial \mu}{\partial \tau}\mathbf{v}_{\mathbf{0}},$$

$$\frac{\partial \mathbf{p}_{\mathbf{v}}(0)}{\partial \mathbf{p}_{\mathbf{v0}}} = \frac{\partial \dot{\mathbf{p}}_{\mathbf{v}}(0)}{\partial \dot{\mathbf{p}}_{\mathbf{v0}}} = \mathbf{E}, \quad \frac{\partial \mathbf{p}_{\mathbf{v}}(0)}{\partial \dot{\mathbf{p}}_{\mathbf{v0}}} = \frac{\partial \dot{\mathbf{p}}_{\mathbf{v}}(0)}{\partial \mathbf{p}_{\mathbf{v0}}} = \frac{\partial \mathbf{p}_{\mathbf{v}}(0)}{\partial \tau} = \frac{\partial \dot{\mathbf{p}}_{\mathbf{v}}(0)}{\partial \tau} = 0.$$

TYPICAL CEV-PROBLEM WITH THRUST SWITCHINGS

Purpose: To minimize $J = \int_0^T \delta \frac{P}{w} dt$ for a dynamical system obeying the differential equations

$$\left.\begin{aligned} \frac{d^2\mathbf{x}}{dt^2} &= \Omega_{\mathbf{x}} + \delta \frac{P}{m}\mathbf{e}, \\ \frac{dm}{dt} &= -\delta \frac{P}{w}, \end{aligned}\right\}$$

and having following boundary conditions: $t = 0$: $\mathbf{x}(0) = \mathbf{x}_0$, $d\mathbf{x}(0)/dt = \mathbf{v}_0$, $m(0) = m_0$, $t = T$: $\mathbf{x}(T) = \mathbf{x}_f$, $d\mathbf{x}(T)/dt = \mathbf{v}_f$.
Here δ is step-like thrusting function, P – thrust, w – exhaust velocity, m – spacecraft mass.

Pontryagin's maximum principle reduces this OCP to the following TPBVP:

$$\left.\begin{aligned} \frac{d^2\mathbf{x}}{dt^2} &= \Omega_{\mathbf{x}} + \delta \frac{P}{m}\frac{\mathbf{p_v}}{p_v}, \\ \frac{dm}{dt} &= -\delta \frac{P}{w}, \end{aligned}\right\} \qquad \left.\begin{aligned} \frac{d^2\mathbf{p_v}}{dt^2} &= -\Omega_{\mathbf{xx}}\mathbf{p_v}, \\ \frac{dp_m}{dt} &= -\delta \frac{P}{m^2} p_v, \end{aligned}\right\}$$

$t = 0$: $\mathbf{x}(0) = \mathbf{x}_0$, $d\mathbf{x}(0)/dt = \mathbf{v}_0$, $m(0) = m_0$, $t = T$: $\mathbf{x}(T) = \mathbf{x}_f$, $d\mathbf{x}(T)/dt = \mathbf{v}_f$, $p_m(T) = 0$, where step-like thrusting function $\delta = \begin{cases} 1, & \text{if } \psi_s > 0, \\ 0, & \text{if } \psi_s \le 0, \end{cases}$

and switching function $\psi_s = \frac{p_v}{m} - \frac{1 + p_m}{w}$

In fact, it is necessary to solve equation $\mathbf{f}(\mathbf{z}) = 0$, where $\mathbf{f}(\mathbf{z}) = \begin{pmatrix} \mathbf{x}(T) - \mathbf{x}_f \\ \mathbf{v}(T) - \mathbf{v}_f \\ p_m(T) \end{pmatrix}$ is vector of residuals, and $\mathbf{z} = \begin{pmatrix} \mathbf{p_x}(0) \\ \mathbf{p_v}(0) \\ p_m(0) \end{pmatrix}$

LP-to-CEV CONTINUATION USING FROZEN TRAJECTORY STRUCTURE

1. Sequence of thrusting and coasting arcs is assumed fixed

2. Equations of optimal motion are following:

(the equations correspond to LP-problem if τ=0 and to CEV-problem if τ=1)

$$\frac{d^2\mathbf{x}}{dt^2} = \Omega_{\mathbf{x}} + \delta \cdot \left[(1-\tau) + \tau \frac{P}{m p_v} \right] \mathbf{p}_{\mathbf{v}},$$

$$\frac{d^2\mathbf{p}_{\mathbf{v}}}{dt^2} = \Omega_{\mathbf{xx}} \mathbf{p}_{\mathbf{v}},$$

$$\frac{dm}{dt} = -\delta \frac{P}{w},$$

$$\frac{dp_m}{dt} = \delta \frac{P}{m^2} p_v.$$

2. Switching function: $\psi = \frac{p_v}{m} - \frac{1+p_m}{w}$

3. Initial conditions:

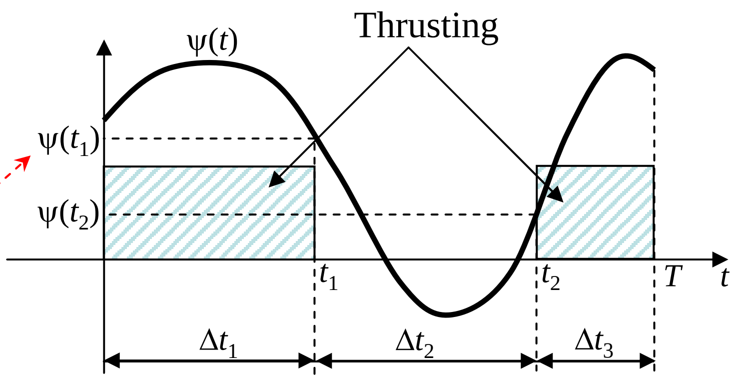
$$\mathbf{x}(0) = \mathbf{x}_0,$$

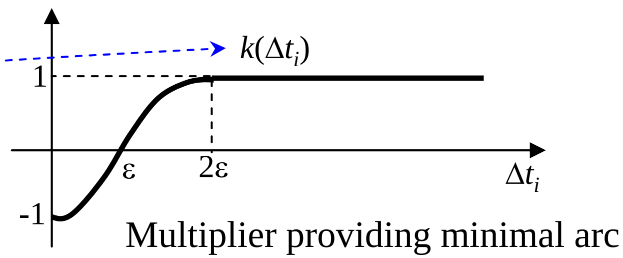
$$\mathbf{v}(0) = \mathbf{v}_0 + V_\infty \frac{\mathbf{p}_{\mathbf{v}}}{p_v},$$

$$m(0) = m_0.$$

4. Final conditions:

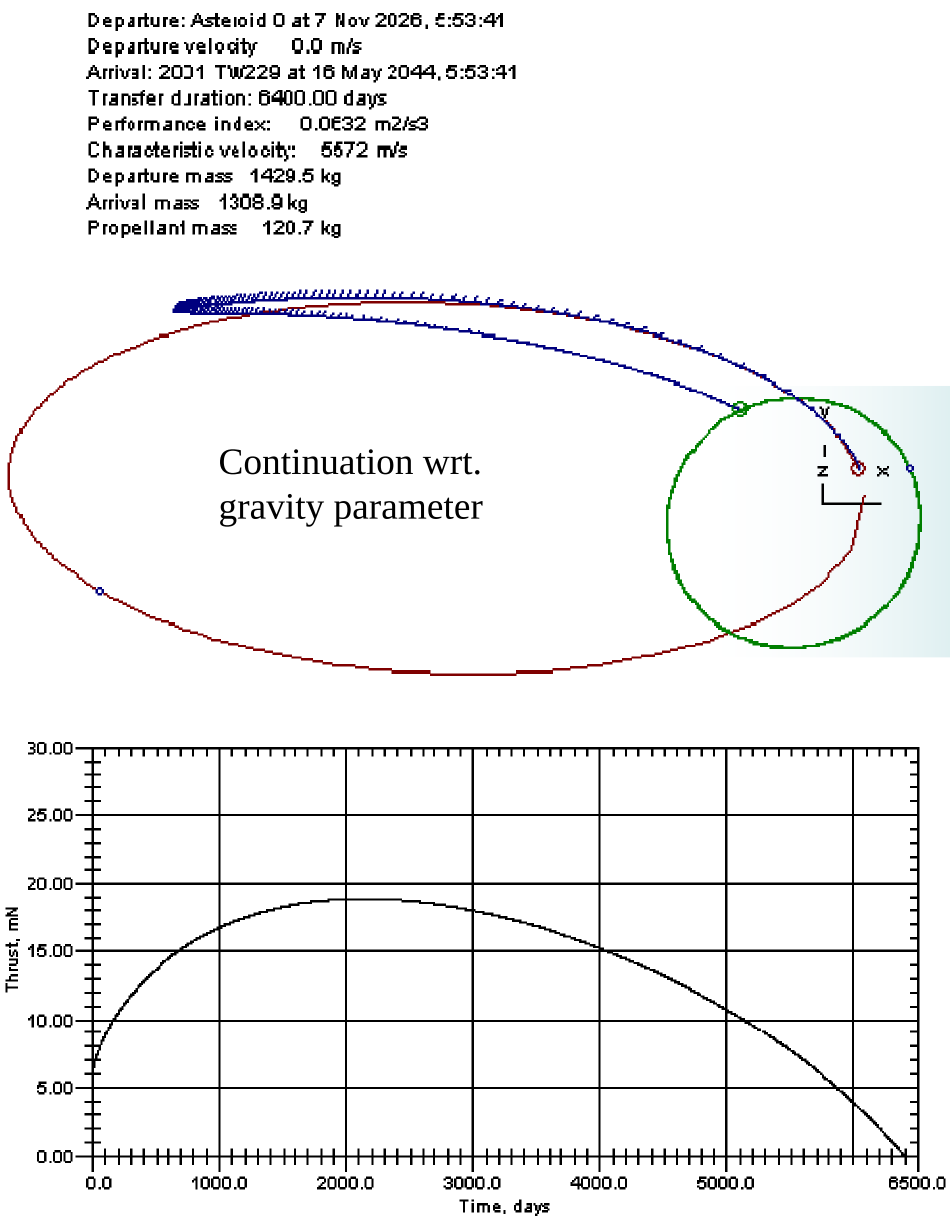
$$\mathbf{f} = \begin{pmatrix} \mathbf{x} - \mathbf{x}_{ast} \\ \mathbf{p}_{\mathbf{v}} - \tau \cdot m \,\mathrm{sgn}\left(\mathbf{v}^{\mathrm{T}} \mathbf{v}_{ast}\right) \mathbf{v}_t \\ p_m \\ \sum_i \Delta t_i - T \\ k(\Delta t_1)\psi(t_1) \\ \vdots \\ k(\Delta t_{N_{arc}-1})\psi(t_{N_{arc}-1}) \end{pmatrix} = 0$$

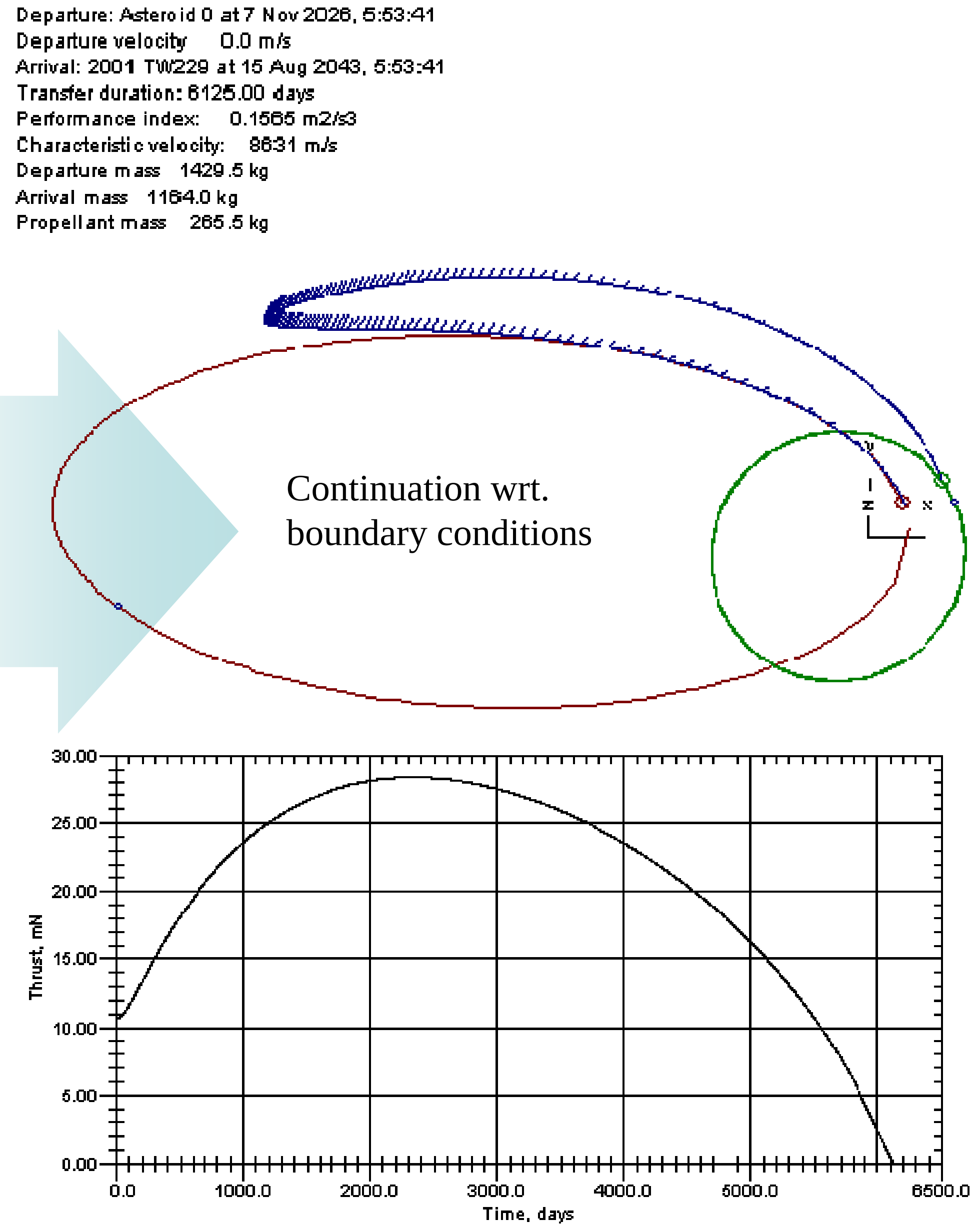




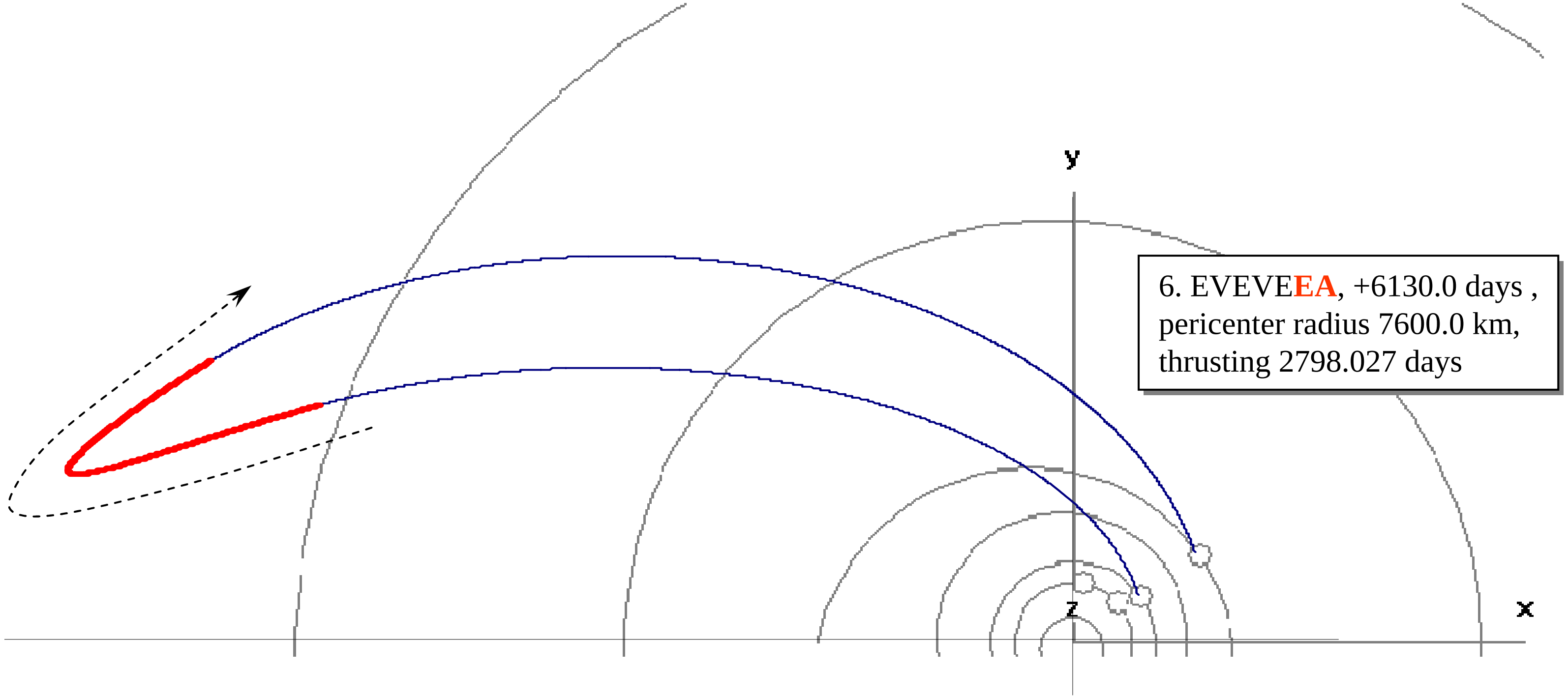
Multiplier providing minimal arc length ε

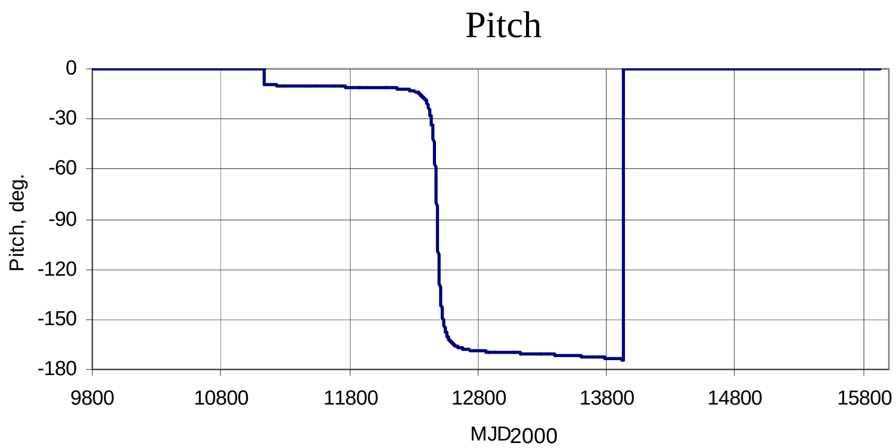
EARTH-to-ASTEROID POWER-LIMITED TRAJECTORY

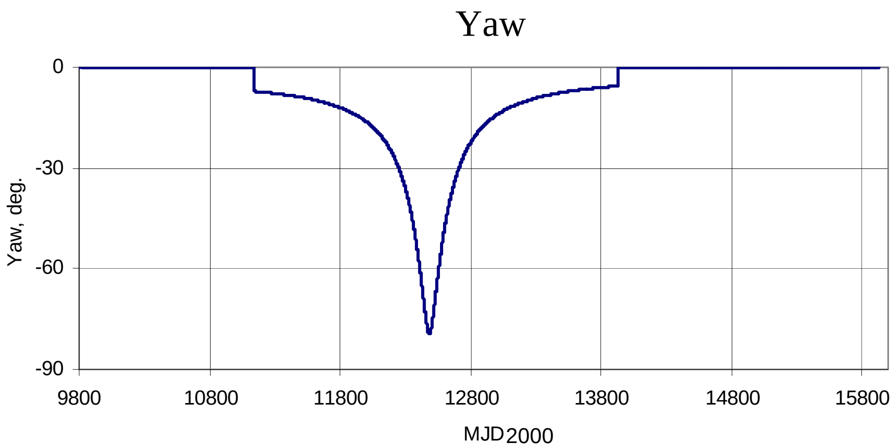




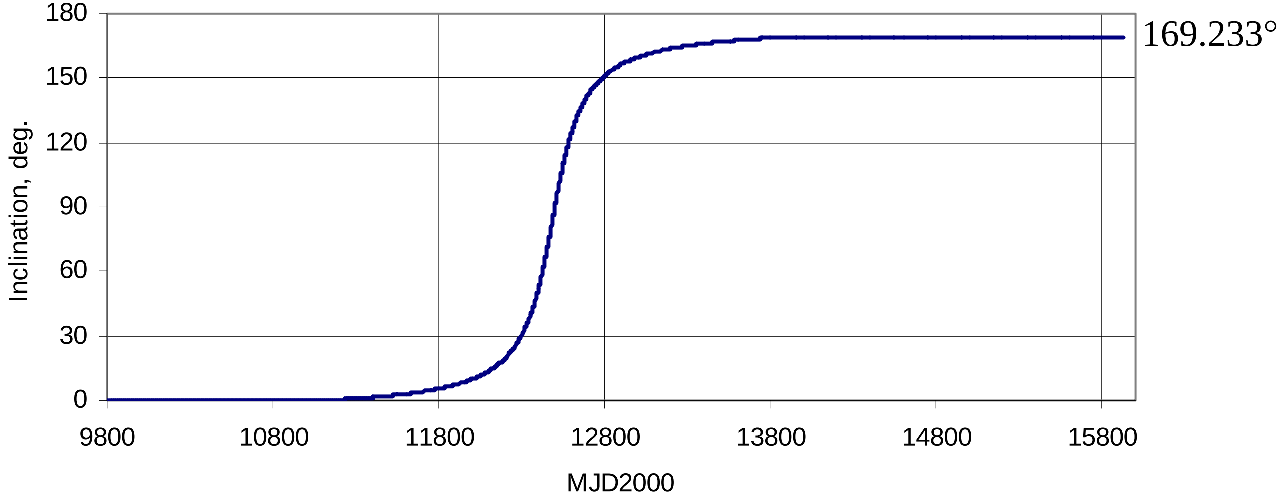
TRAJECTORY DESCRIPTION: FINAL (CEV) EARTH-to-ASTEROID TRAJECTORY

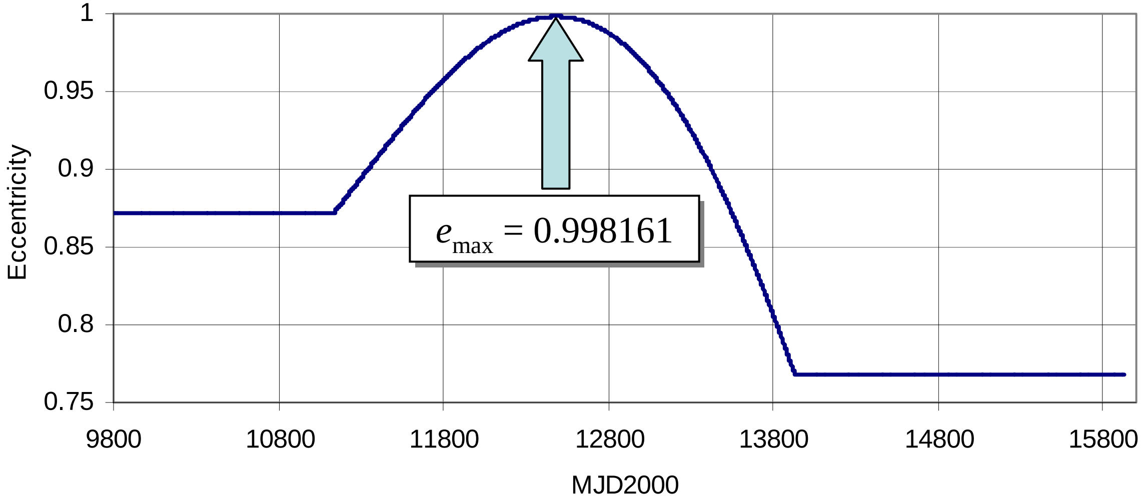






FINAL (CEV) EARTH-to-ASTEROID TRAJECTORY





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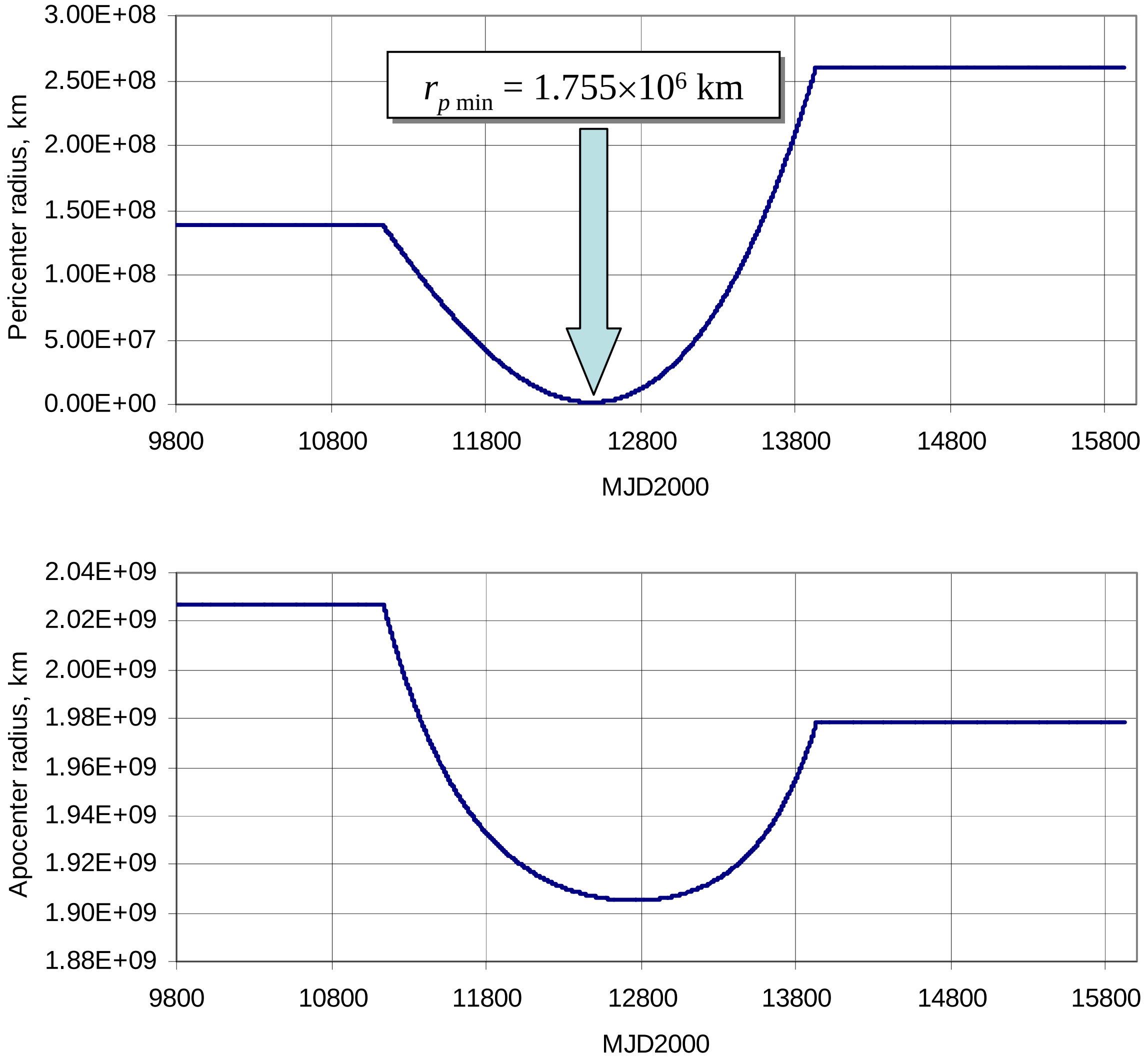
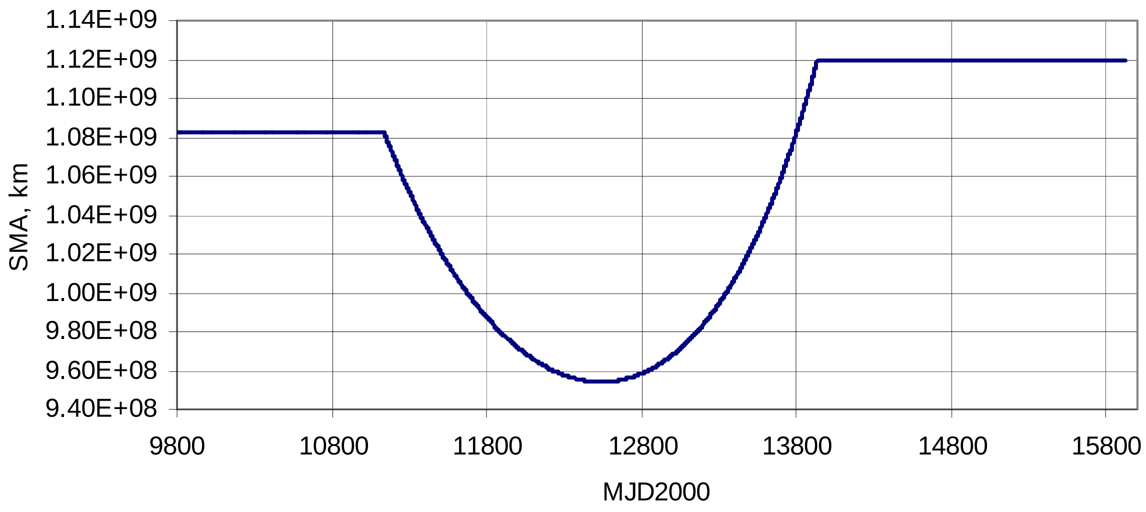
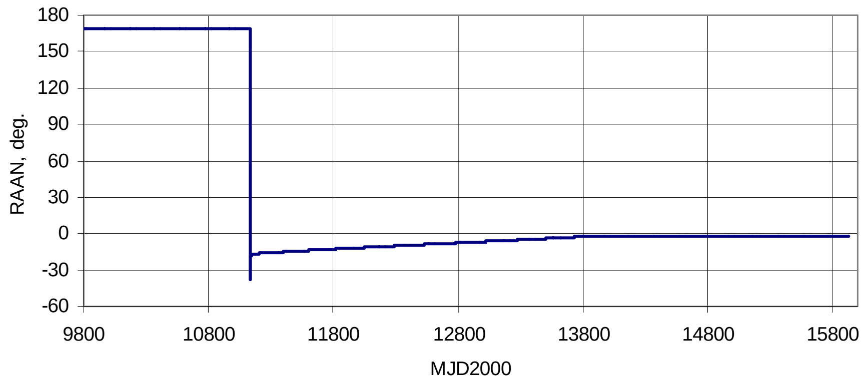
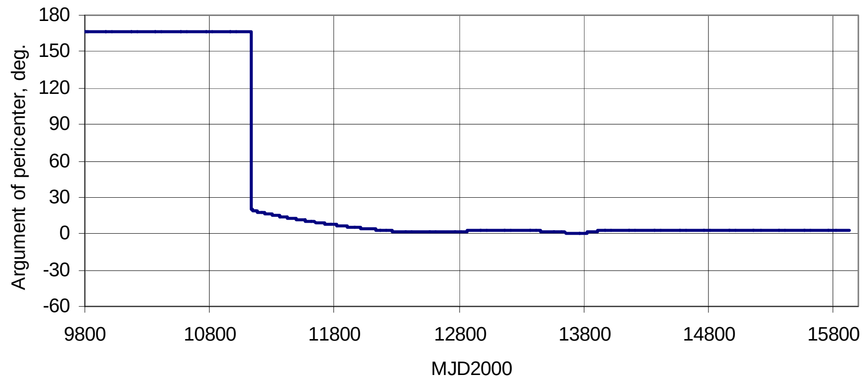


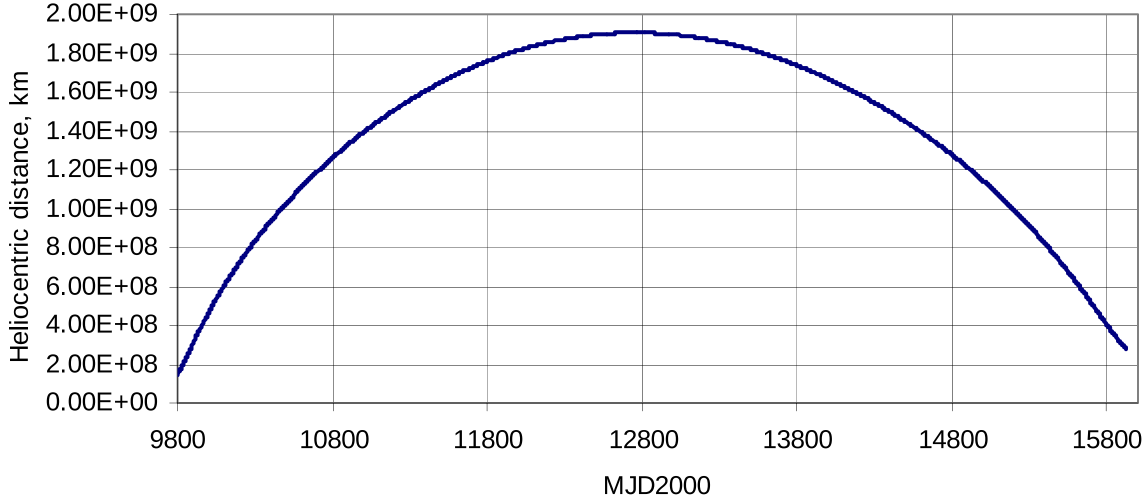
FINAL (CEV) EARTH-to-ASTEROID TRAJECTORY

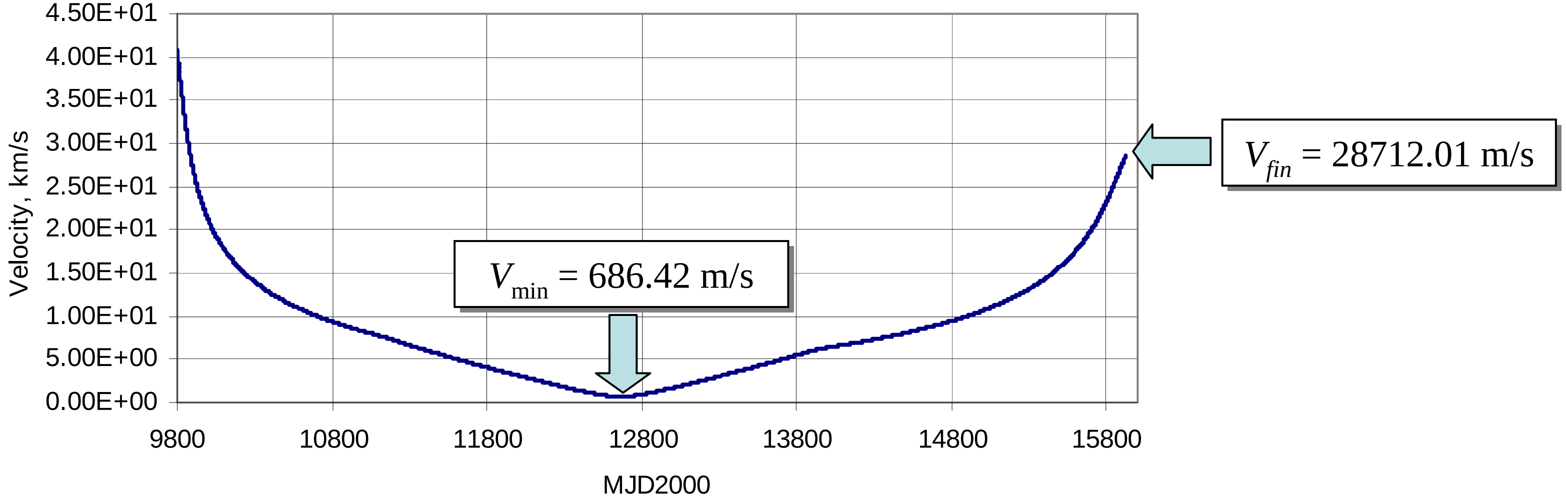






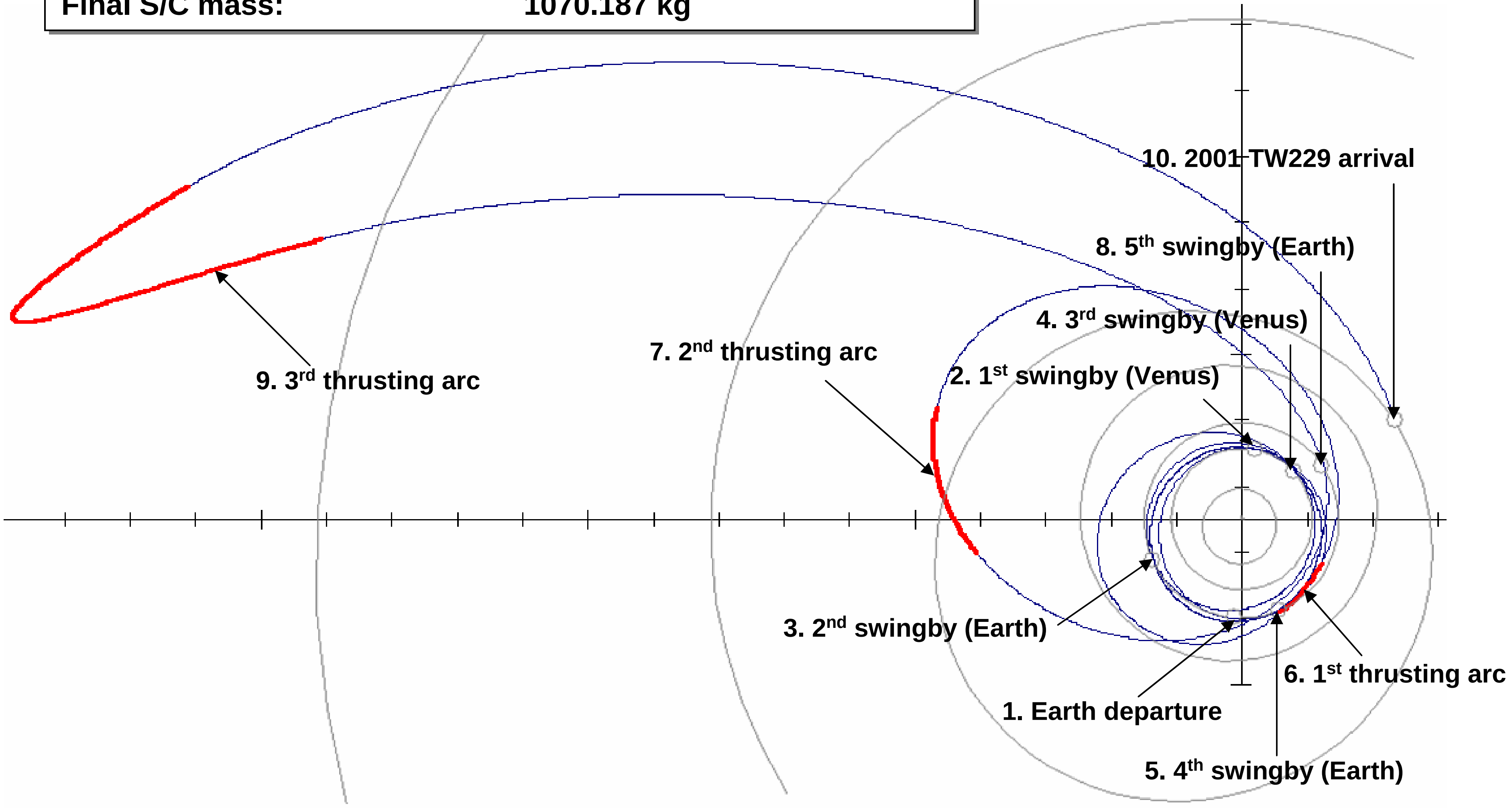
FINAL (CEV) EARTH-to-ASTEROID TRAJECTORY



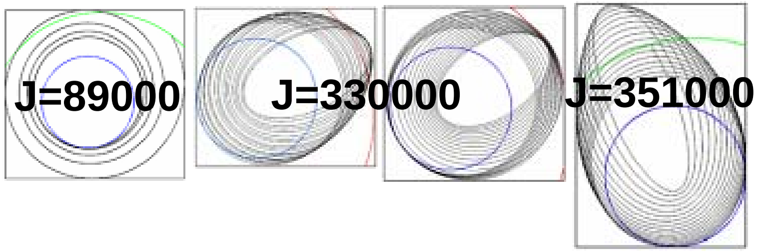


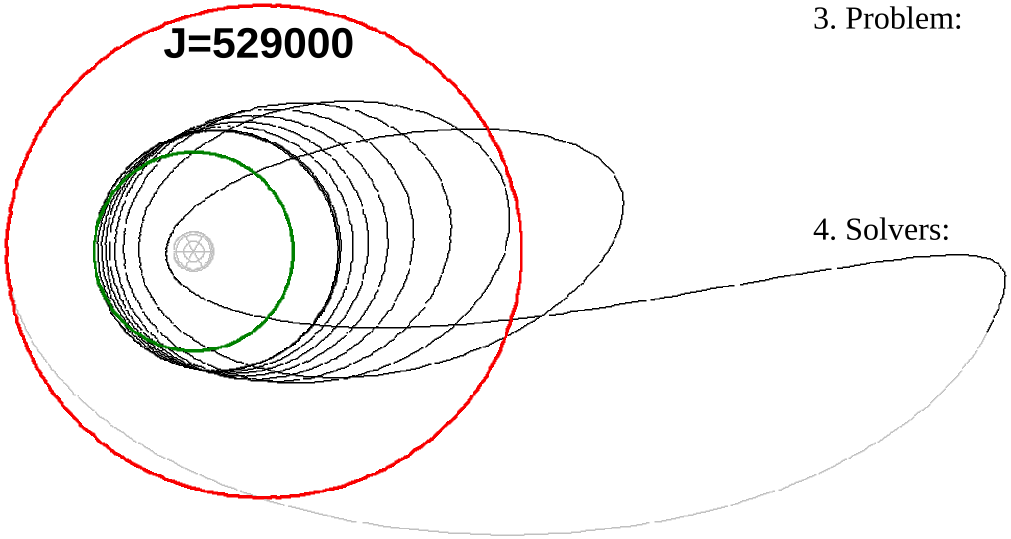
COMPLETE TRAJECTORY

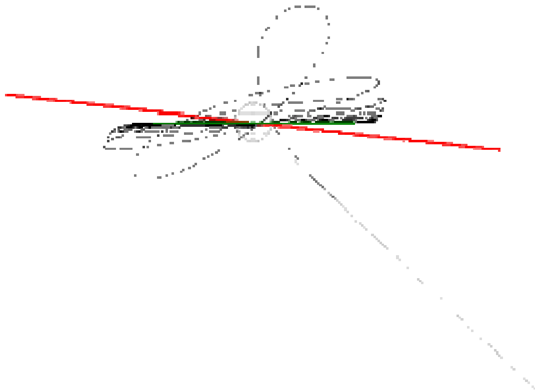
Objective function value:	**1364042.86 kg·km²/sec²;**
Route:	**EVEVEEA**
Launch date:	**JD 2457556.7 (June 17.2, 2016)**
Escape velocity:	**2.474133 km/sec**
Total duration:	**9914.034 days**
Final S/C mass:	**1070.187 kg**



EXAMPLE OF LOCAL OPTIMIZATION COMPLEXITY: DIRECT EARTH-to-ASTEROID TRAJECTORY







1. Initial S/C orbit: line of apsides along to asteroid's line of apsides;
   pericenter radius equals to earth orbit radius at departure date;
   apocenter radius corresponds to asymptotic velocity 2.5 km/s;
   inclination equals to 0.

2. Final S/C orbit: line of apsides along to asteroid's line of apsides;
   pericenter radius equals to asteroid's pericenter radius;
   inclination and apocenter radius are varied.

3. Problem: minimum-time transfer to the final orbit with constrained minimal heliocentric distance (0.2 AU). The constraint is regulated by number of orbits (continuation wrt. gravity parameter), final inclination, and final apocenter radius.

4. Solvers:
   a) Averaged optimal control problem (maximum principle, continuation technique, E-ProTO software).
   b) Unaveraged optimal control problem (maximum principle, continuation technique, averaged solution as an intial guess, E-ProTO software)

DIRECT EARTH-to-ASTEROID TRAJECTORY: RESULTS

INITIAL ORBIT: 1.0 × 1.4239976 AU, i = 0°

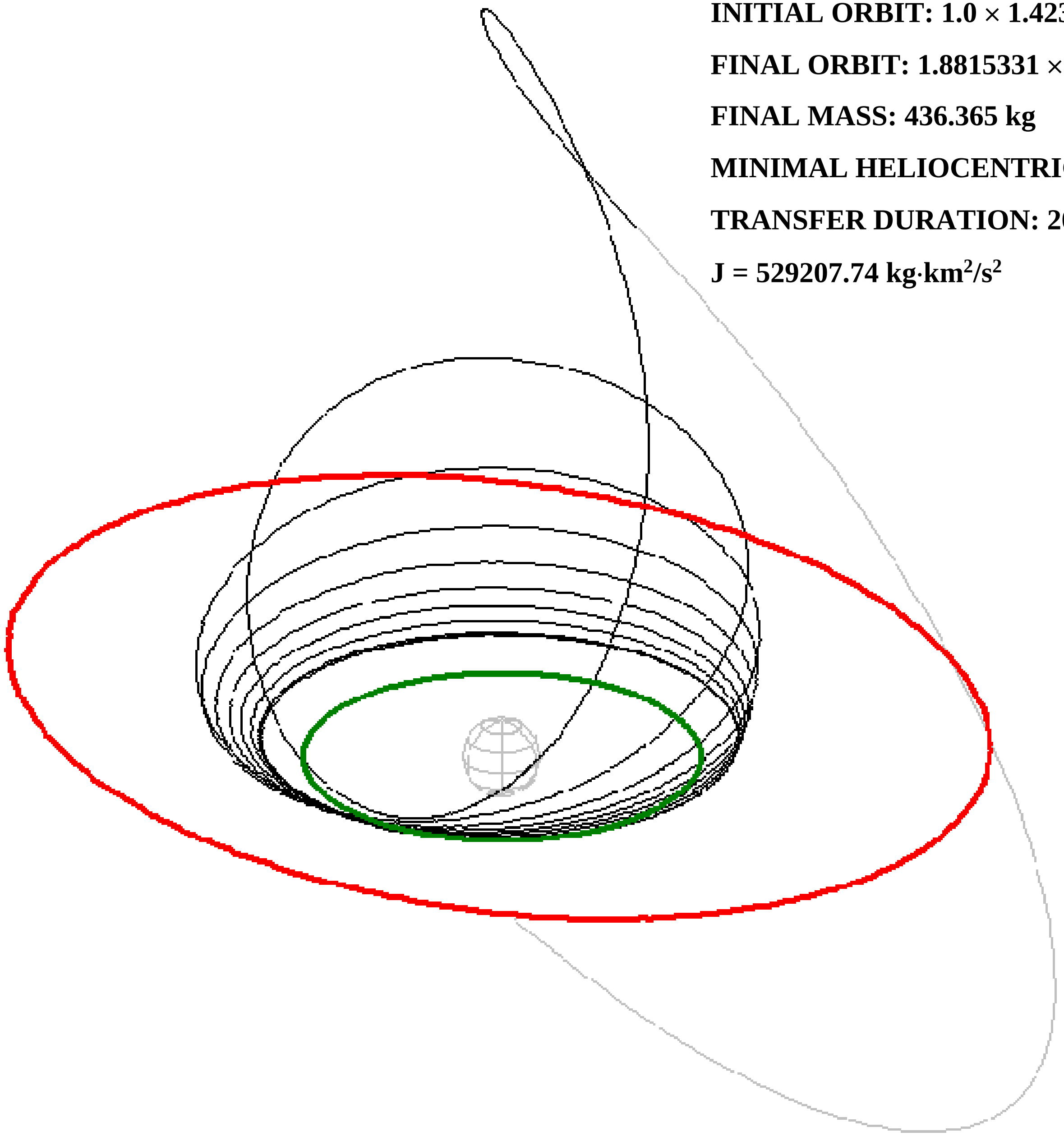
FINAL ORBIT: 1.8815331 × 8.1846558 AU, i = 136.5°

FINAL MASS: 436.365 kg

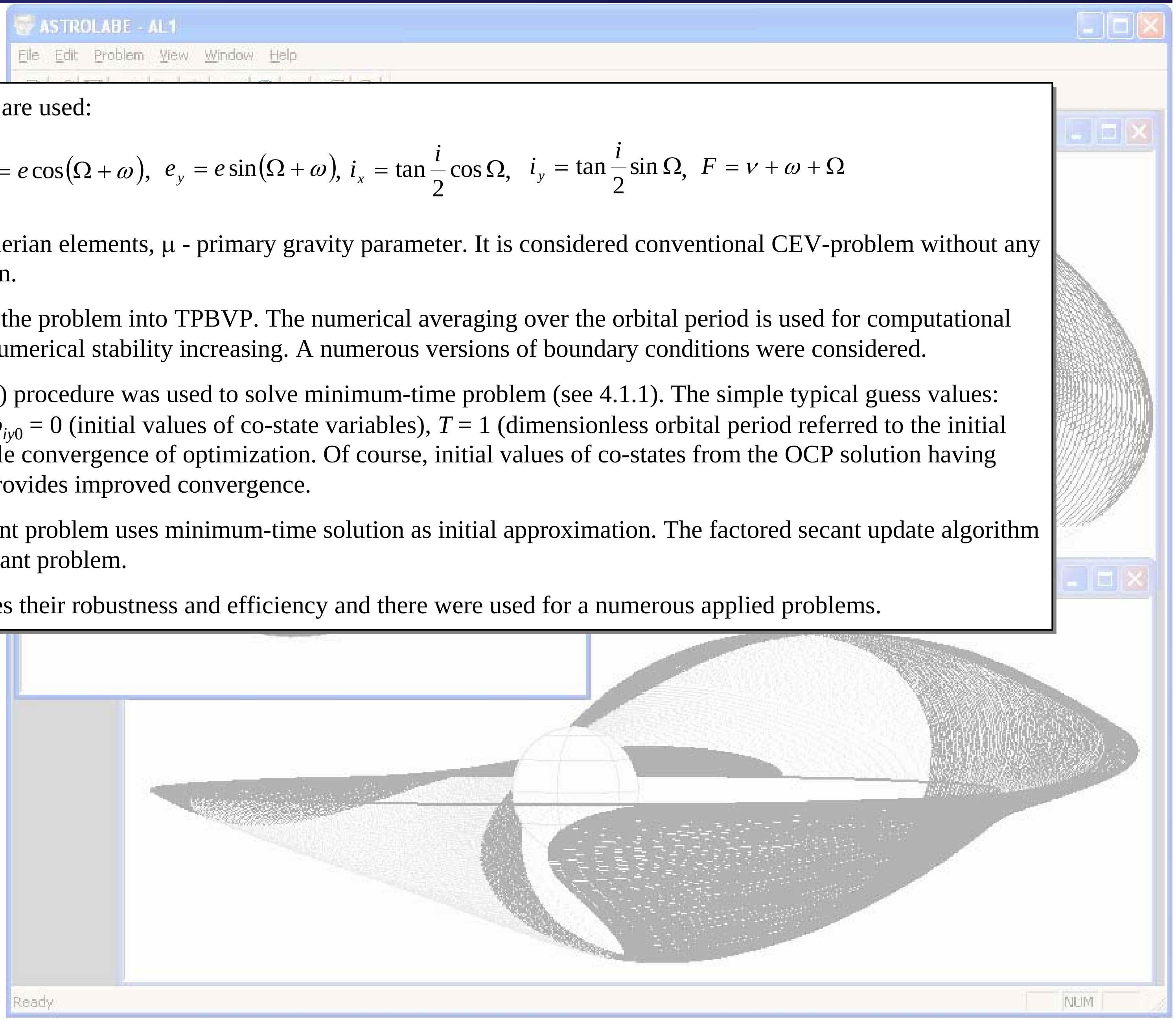
MINIMAL HELIOCENTRIC DISTANCE: 0.2009528 AU

TRANSFER DURATION: 20.658 (thrusting) + 4.784 (coasting) = 25.442 years

J = 529207.74 kg·km^2/s^2



TECHNIQUES OF MULTIREVOLUTIONAL OPTIMIZATION



Equinoctial orbital elements are used:

$$h = \sqrt{\frac{p}{\mu}}, \quad e_x = e\cos(\Omega + \omega), \quad e_y = e\sin(\Omega + \omega), \quad i_x = \tan\frac{i}{2}\cos\Omega, \quad i_y = \tan\frac{i}{2}\sin\Omega, \quad F = \nu + \omega + \Omega$$

Here p, e, ω, ν, i, Ω are Keplerian elements, μ - primary gravity parameter. It is considered conventional CEV-problem without any constraints on thrust direction.

Maximum principle reduces the problem into TPBVP. The numerical averaging over the orbital period is used for computational consumption reducing and numerical stability increasing. A numerous versions of boundary conditions were considered.

The continuation (homotopy) procedure was used to solve minimum-time problem (see 4.1.1). The simple typical guess values: $p_{h0} = \pm 1$, $p_{ex0} = p_{ey0} = p_{ix0} = p_{iy0} = 0$ (initial values of co-state variables), $T = 1$ (dimensionless orbital period referred to the initial orbit) as a rule provides stable convergence of optimization. Of course, initial values of co-states from the OCP solution having close boundary conditions provides improved convergence.

Solver of minimum-propellant problem uses minimum-time solution as initial approximation. The factored secant update algorithm is used for minimum-propellant problem.

Both techniques demonstrates their robustness and efficiency and there were used for a numerous applied problems.

CONCLUSION

1. Tolerable objective function value was obtained without using Jupiter/Saturn flybys
2. Global optimization should be supported by reliable methods of local optimization
3. Continuation technique allows to find “global” minimum among local minimums depending on restricted number of parameters (boundary conditions, transfer duration, number of orbits)
4. THANK YOU FOR ATTENTION