Global Optimization Competition Workshop

Team 4

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Summary

- trajectory optimization at Politecnico di Torino
- what the hell of a problem is this ?
- some help from astrodynamics
- envisaged solution
- legs optimization
- joining the legs
- eureka !
- what's next ?

Trajectory Optimization at Politecnico di Torino

(1)

- indirect method based on optimal control theory (OCT)
- state equations

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \boldsymbol{V} \qquad \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = \boldsymbol{g} + \frac{\boldsymbol{T}}{m} \qquad \frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{T}{c}$$

- Hamiltonian

$$H = \boldsymbol{\lambda}_r^T \boldsymbol{V} + \boldsymbol{\lambda}_V^T \boldsymbol{g} + T S_F$$

- switching function

$$S_F = \boldsymbol{\lambda}_V^T \boldsymbol{T}/(mT) - \lambda_m/c$$

Trajectory Optimization at Politecnico di Torino (2)

- controls maximize H in agreement with Pontryagin's maximum principle (PMP)
  - thrust parallel to $\boldsymbol{\lambda}_V$ (primer vector)
  - maximum thrust when $S_F = \lambda_V/m - \lambda_m/c > 0$
  - zero thrust when $S_F < 0$

Trajectory Optimization at Politecnico di Torino (3)

- trajectory split into arcs:
  - thrust arcs (T)
  - coast arcs (C)
  - flybys (F)
- switching structure (i.e., succession of arcs) fixed in advance
- state or control variables discontinuous at the arc junctions
- boundary conditions at the arc extremities
- OCT provides Euler-Lagrange equations for the adjoint variables and additional boundary conditions at the arc junctions

Trajectory Optimization at Politecnico di Torino (4)

- mission constraints and OCT define a multipoint boundary value problem
- problem parameters
  - departure and arrival dates
  - dates of engine switches (on/off)
  - flyby dates
  - initial velocity, position, and adjoint variables
  - velocity and adjoint variables soon after flybys
- tentative values are assumed
- Newton's method to obtain convergence
- switching structure changed when PMP is violated

What the Hell of a Problem Is This ?

Proposed Problem

- search for global optimum
- new mission concept: no solution available
- long trip time: large number of flybys
- ballistic arcs prevail; limited use of thrust

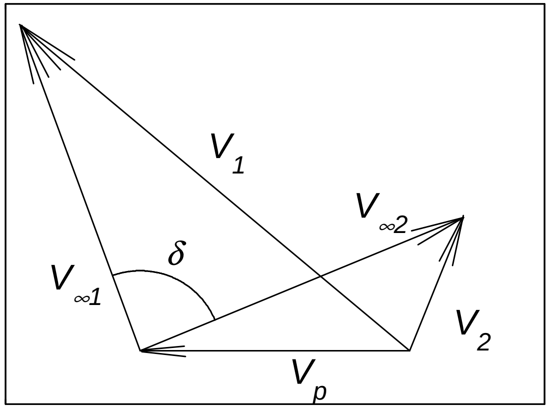
Indirect Optimization

- finds local optima
- requires tentative solutions
- no procedure to asses flyby succession
- accurate thrust program optimization

- give up the search for the global optimum
- search for a simple and locally optimal trajectory with few flybys, which fits to the procedure

Some Help from Astrodynamics (1)

- a retrograde orbit must be sought
- gravity assist from a giant planet can reverse the spacecraft velocity
  - the hyperbolic excess velocity (V_∞) must be larger than the planet's velocity
  - a large radial velocity component is preferable when a limited rotation δ is allowed



Some Help from Astrodynamics (2)

- gravity assist from Earth and Venus can be used to reach Jupiter or Saturn
- the same heliocentric energy is obtained with the lowest hyperbolic excess velocity if the latter is parallel to the planet’s velocity
- for the same energy the radial velocity at the giant planet encounter is larger when the spacecraft arrives from Venus than arriving from the Earth
- VJA and VSA legs are considered

Some Help from Astrodynamics (3)

- a large hyperbolic excess velocity at Venus is required
- Venus must be reached coming from a large aphelion
- Earth gravity assist can be used to increase the orbit energy and aphelion
- the departure hyperbolic excess velocity (2.5 km/s) can insert the spacecraft into an orbit with a 1.33-year period

Envisaged Solution

- 3:4 ΔV-EGA trajectory: the spacecraft performs three revolutions around the sun; thrust is used to encounter the Earth again after 4 years with a larger V_∞
- Earth gravity assist increases the orbit energy and aphelion and inserts the spacecraft into an Earth-Venus transfer
- Venus is reached with a large hyperbolic excess velocity and a Venus flyby is performed to insert the spacecraft into a Venus-Jupiter (or Saturn) transfer
- giant planet flyby to make the orbit retrograde and intercept the asteroid

Legs Optimization - from Venus to 2001 TW229 (1)

- Venus-Jupiter (or Saturn)-2001 TW229 transfer
  - the actual index J (change in asteroid energy) is maximized while assigning $V_\infty = 15$ km/s (sufficient to reach the giant planets) when leaving Venus
  - planets and asteroid positions are left free
  - free-height flyby
  - a ballistic trajectory is initially considered (C-F-C)
  - easy convergence
  - PMP to determine the optimal switching structure: a 4-arc (T-F-T-C) structure is found
  - minimum-height constraint added for Jupiter flyby

Legs Optimization - from Venus to 2001 TW229 (2)

- solutions can be flown every venusian year
- actual positions of Jupiter (or Saturn) and asteroids on the possible flyby and arrival dates are compared to the flyby and arrival positions of the optimal solution
- mission opportunities when the differences are low
- good Venus-Jupiter departure (from Venus) dates
  - 10/10/2028 (too early)
  - 29/01/2041

Legs Optimization - from Venus to 2001 TW229 (3)

- actual Jupiter and asteroid positions taken into account
- a feasible leg is computed
    - Venus departure 23/01/2041
    - Jupiter flyby 22/03/2042
    - 2001 TW229 arrival 26/09/2047
- the corresponding (optimal) position for Venus flyby is found $\vartheta_V \approx 90$ deg

Legs Optimization - from Venus to 2001 TW229 (4)

- possible Venus-Saturn departure dates
    - 11/03/2050 asteroid not in the right place
    - 22/10/2050 asteroid not in the right place
    - 04/06/2051 arrival beyond 2060
- Saturn flyby not investigated further due to lack of time

Legs Optimization - from Earth to Venus (1)

- 3:4 ΔV-EGA trajectory
  - the final mass is maximized while assigning $V_\infty = 8$ km/s at the Earth encounter (this value is sufficient to reach a 3.5 AU aphelion)
  - departure date fixed; departure position known; v_∞=2.5 km/s parallel to Earth's velocity
  - three-arc structure (coast-thrust-coast); short thrust arc at the first aphelion passage
  - easy convergence
  - PMP to determine the optimal switching structure (thrust arcs are progressively added where S_F is positive): 12 arcs (T-C-T-C-T-C-T-C-T-C-T-C) are required

Legs Optimization - from Earth to Venus (2)

- Earth flyby-Venus transfer
  - the final mass is maximized while assigning $V_\infty = 8$ km/s when leaving the Earth and $V_\infty = 15$ km/s at Venus encounter
  - Earth's position is fixed
  - Venus position is left free
  - a 1-rev 3-arc structure does not allow convergence
  - a 2-rev 3-arc structure is assumed (thrust arc at the first aphelion passage)
  - convergence rather easy
  - PMP to determine the optimal switching structure: a 5-arc (C-T-C-T-C) structure is found

Legs Optimization - from Earth to Venus (3)

- from Earth to Venus with Earth gravity assist (EEV)
  - maximum final mass with $V_\infty = 15$ km/s at Venus encounter (V_∞ free at minimum-height Earth flyby)
  - initial Earth position fixed
  - Venus position free
  - convergence is straightforward using tentative values from the previous solutions
- many trajectories with similar performance are easily obtained by changing the departure position
- Earth's initial position is optimized and the arrival position fixed at the value required by the VJA leg (Venus position is still left free)

Joining the Legs - Venus Flyby(s)

- the angle between the arrival and departure V_∞ at Venus is about twice the maximum allowable value
- two Venus flybys (same position) are required
  - first: partial rotation, s/c inserted into an orbit that encounters Venus again
  - second: rotation completed, departure to Jupiter
- ballistic Venus-Venus transfer
- time between flybys multiple of the period of Venus orbit
- a 19-venusian-year period satisfy all the constraints (18 or 20 may also be considered)

Joining the Legs - from Earth to Venus

- Earth-Venus trajectories can be flown every year
- actual positions of Venus on the possible arrival dates compared to the arrival position of the optimal solution
- mission opportunities when the difference is low
- a good opportunity with arrival 19 venusian years before the second flyby of Venus is found
- Venus position is fixed and a feasible leg is computed
  - Earth departure 29/01/2019
  - Venus arrival 17/05/2029

Joining the Legs

Solution for Constant Orbital Parameters

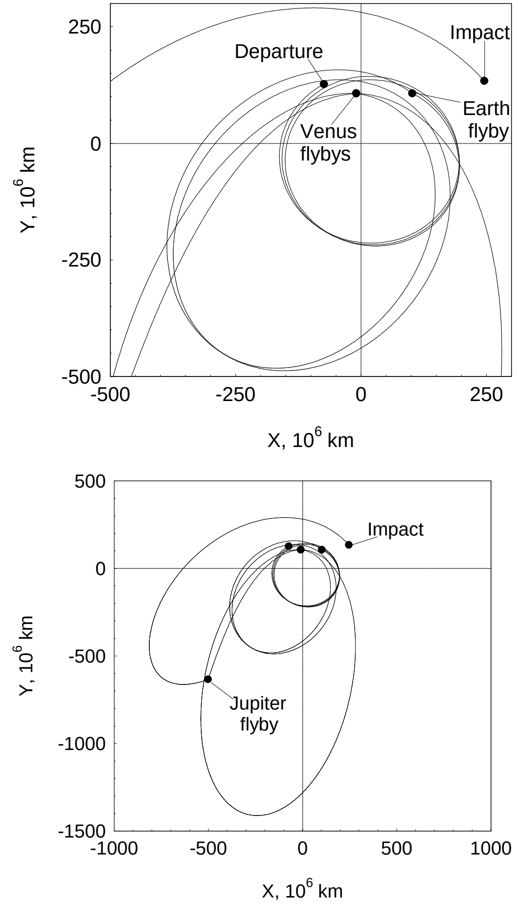
- Earth-Venus transfer with Earth gravity assist
- single Venus flyby
  - periapsis height reduced to make acceptable a rotation twice the allowable value (for $V_\infty = 15$ km/s)
  - 19-venusian-year time discontinuity
- Venus-2001 TW229 transfer with Jupiter gravity assist
- easy convergence
- periapsis height of Venus flyby adjusted to make feasible the flyby splitting and the insertion into the 19-venusian-year Venus-Venus transfer

Eureka (1)

- JPL ephemeris are progressively introduced replacing the orbital parameters formulation
- Earth and Jupiter: no sensible change in the solution
- Venus: thrust must be used to intercept Venus again for the second flyby
- the complete trajectory is now computed
- two Venus flybys with the actual constraints
- difficult convergence: a particular procedure must be used
  - very small thrust first assumed in the Venus-Venus leg
  - thrust progressively increased
  - change in the switching structure after the second flyby are introduced according to PMP

Eureka (2)

- EVVJA mission obtained
  - departure 20/01/2019
  - arrival 26/09/2047
  - switching structure: T-C-T-C-T-C-T-C-T-C-T-C Earth flyby C-T-C-T-C Venus flyby C-T-C Venus flyby T-C-T Jupiter flyby T-C
  - 29 arcs, 74 parameters
  - very short thrust arc (12 hr) between Venus flybys



What's Next

- a single solution is a "gold mine" when using indirect methods
- other trajectories can be found to improve the performance index
  - different launch windows (e.g., departure date)
  - different flyby structures (e.g., Saturn instead of Jupiter)
  - more complex missions (e.g Jupiter-Saturn-Jupiter flyby sequence)